A new construction of indecomposable solutions of the Yang-Baxter equation

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Indecomposable solutions of the Yang-Baxter equation

A caratterization of indecomposable cycle sets

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Definition

Let X be a non-empty set, $r : X \times X \to X \times X$ a map and write

$$r(x, y) = (\lambda_x(y), \rho_y(x))$$

for all $x, y \in X$.

Then (X, r) is said to be a non-degenerate involutive set-theoretic solution of the Yang-Baxter equation if and only if the following properties hold:

(1) $r^2 = id_{X^2}$; (r is involutive)

(2) $\lambda_x, \rho_y \in Sym_X$ for all $x \in X$; (r is non degenerate)

(3) $r_1 r_2 r_1 = r_2 r_1 r_2$

where $r_1 = r \times id_X$ and $r_2 = id_X \times r$.

Convention: From now on, by a solution I mean a non-degenerate involutive set-theoretic solution.

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Rump (2005) showed that there is a bijective correspondence between the solutions of the Yang-Baxter equation and a particular algebraic structure: **the left non-degenerate cycle sets**.

Definition (Rump, 2005)

A *left cycle set* is a set X with a binary operation $\cdot$ such that the left multiplication $\sigma_x : X \longrightarrow X,\ y \longmapsto x \cdot y$ is bijective, and the equation

$$(x\cdot y)\cdot(x\cdot z) = (y\cdot x)\cdot(y\cdot z) \quad (1)$$

holds for all $x, y, z \in X$.

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Therefore, if (X, r) is a solution, where $r(x, y) := (\lambda_x(y), \rho_y(x))$ then $(X, \cdot)$ is its **left cycle set associated** defined by

$$\sigma_x := \lambda_x^{-1}$$

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for every $x \in X$.

Vice versa if $(X, \cdot)$ is a left non-degenerate cycle set and σ_x its left multiplication then, for all $x, y \in X$

$$r(x, y) := (\sigma_x^{-1}(y), \sigma_x^{-1}(y) \cdot x)$$

is its **associated solution**.

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Example

Let X be a set. If $\lambda_x = \rho_x = id_X$ for all $x \in X$ then

$$r(x, y) := (y, x)$$

for all $x, y \in X$ is a solutions of the Yang Baxter equation and it is called *the trivial solutions*.

Example

Let X be a set, the non-degenerate left cycle sets $(X, \cdot)$ associated to the trivial solutions is defined by $\sigma_x = \lambda_x^{-1} = id_X$, i.e.

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Definition (P. Etingof, T.Schedler, A.Soloviev, 1999)

A solution (X, r) is said to be *decomposable* if there exists a non trivial partition $X = Y \cup Z$ with $r(Y \times Y) \subseteq Y \times Y$ and $r(Z \times Z) \subseteq Z \times Z$, such that the restrictions of r to $Y \times Y$ and $Z \times Z$ are again non-degenerate.

Otherwise it is said *indecomposable*.

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What about cycle sets associated to indecomposable solutions?

The definition of the decomposable solutions corresponds to the definition of the decomposable non-degenerate left cycle sets and vice versa.

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The definition of the decomposable solutions corresponds to the definition of the decomposable non-degenerate left cycle sets and vice versa.

Definition (Rump, 2005)

A non-degenerate left cycle set X is said to be *decomposable* if it is an union of two disjoint subset $X = Y \cup Z$, such that $\sigma_x(Y) \subseteq Y$ for all $x \in Y$ and $\sigma_x(Z) \subseteq (Z)$, for all $x \in Z$.
Otherwise it is called *indecomposable*.

Studying indecomposable solutions means studying indecomposable non-degenerate left cycle sets.

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Why is it important to study indecomposable non-degenerate left cycle sets and so the associated indecomposable solutions?

Because from indecomposable non-degenerate left cycle sets, by some constructions, you can obtain new non-degenerate left cycle sets.

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Example

Let $Y = \{1, 2\}$ be the indecomposable left cycle sets defined by

$\cdot$	1	2
1	2	1
2	2	1

and let $Z = \{3\}$ be the trivial left cycle sets.
Let's try to construct the left cycle set $X = Y \cup Z$.
Let's get togheter the two disjoint cycle sets and define a new operation on X such that X is a left cycle set. For example:

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Example

$\cdot$	1	2	3
1	2	1	3
2	2	1	3
3	1	2	3

You can only construct another one

$\cdot$	1	2	3
1	2	1	3
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Problem

How is it possible to construct indecomposable non-degenerate left cycle sets?

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Definition

Let X be a non-degenerate left cycle set, and $\sigma : X \to Sym_X$, $x \mapsto \sigma_x$. We denote by $\mathcal{G}(X)$ the subgroup of Sym_X generated by the image $\sigma(X)$ of σ.

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In literature it is known that

Proposition (P. Etingof, T.Schedler, A.Soloviev, 1999)

A non-degenerate left cycle set X is indecomposable if and only if the associated permutation group $\mathcal{G}(X)$ is transitive on X.

Theorem (Rump, 2005)

Every square-free non-degenerate left cycle set is decomposable.

Recall that a non-degenerate left cycle set is *square-free* if $\mathfrak{q} : X \to X,\ x \mapsto x \cdot x$ is such that $\mathfrak{q} = id_X$.

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Proposition (P. Etingof, T.Schedler, A.Soloviev, 1999)

If X is finite and $|X| = p$ with p a prime number. Every indecomposable non-degenerate left cycle set , up to isomorphism, is defined by $x \cdot y := \alpha(y)$ where $\alpha := (12 \ldots p) \in Sym_X$.

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Example

Let X be the cycle set defined by

$\cdot$	1	2	3
1	2	3	1
2	2	3	1
3	2	3	1

Then X is indecomposable.

Moreover if $|X| \leq 8$, by calculation, all indecomposable cycle sets have been found.

Problem

What about cycle sets of cardinality greater then 8?

A large family of indecomposable non-degenerate left cycle sets is obtained by Marco Castelli, Francesco Catino and G.P.

Let A, B be non trivial sets and I a left cycle set, $\beta : A \times A \times I \longrightarrow Sym(B)$ and $\gamma : B \longrightarrow Sym(A)$.

Put $\beta_{(a,b,i)} := \beta(a, b, i)$, $\gamma_a := \gamma(a)$ and let $\cdot$ be the operation on $A \times B \times I$ given by

$$(a, b, i)\cdot (c, d, j) := \begin{cases} (c,\ \beta_{(a,c,i)}(d),\ i\cdot j), & \text{if } i = j \\ (\gamma_b(c),\ d,\ i\cdot j), & \text{if } i \neq j \end{cases}. \qquad (2)$$

Then we write $X(A, B, I, \beta, \gamma)$ to indicate $(A \times B \times I, \cdot\,)$

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Theorem (M. Castelli, F. Catino, G.P., 2017)

Assume that for $X(A, B, I, \beta, \gamma)$

1) $\gamma_a \gamma_b = \gamma_b \gamma_a$,
2) $\beta_{(a,c,i)} = \beta_{(\gamma_b(a), \gamma_b(c), j \cdot i)}$,
3) $\gamma_{\beta_{(a,c,i)}(d)} \gamma_b = \gamma_{\beta_{(c,a,i)}(b)} \gamma_d$,
4) $\beta_{(a,c,i \cdot i)} \beta_{(b,c,i)} = \beta_{(b,c,i \cdot i)} \beta_{(a,c,i)}$.

hold for all $a \in A$, $b, c \in B$, $i, j \in I$, $i \neq j$. *Then* $X(A, I, \beta, \gamma)$ *is a non-degenerate left cycle set.*

Theorem (M. Castelli, F. Catino, G.P., 2017)

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hold for all $a \in A$, $b, c \in B$, $i, j \in I$, $i \neq j$. *Then* $X(A, I, \beta, \gamma)$ *is a non-degenerate left cycle set.*

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Theorem (M. Castelli, F. Catino, G.P., 2017)

Assume that for $X(A, B, I, \beta, \gamma)$

1) $\gamma_a\gamma_b = \gamma_b\gamma_a$,
2) $\beta_{(a,c,i)} = \beta_{(\gamma_b(a),\gamma_b(c),j\cdot i)}$,
3) $\gamma_{\beta_{(a,c,i)}(d)}\gamma_b = \gamma_{\beta_{(c,a,i)}(b)}\gamma_d$,
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Now we give a characterization of the indecomposable non-degenerate left cycle sets $X := X(A, B, I, \beta, \gamma)$.

Definition

For every $i \in I$ let H_i be the subgroup of $\mathcal{G}(X)$ given by

$$H_i := \{h \in \mathcal{G}(X) | h(a, b, i) \in A \times B \times \{i\} \forall a \in A, b \in B\}.$$

Proposition (M. Castelli, F. Catino, G.P., in preparation)

X is indecomposable if and only if I is indecomposable and H_i is transitive on $A \times B \times \{i\}$ for every $i \in I$.

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Example

Let $A = B := \mathbb{Z}/2\mathbb{Z}$, $I := \{1, 2, 3\}$ be the indecomposable non-degenerate left cycle set defined by $\sigma_1 = \sigma_2 = \sigma_3 := (123)$, $\gamma_a := t_{-a-1}$ for every $a \in A$ and

$$\beta_{(a,a,i)} = id_A \qquad \beta_{(a,b,i)} := (12)$$

for all $a, b \in A$, $a \neq b$, $i \in I$.

$$H_i =< \{\sigma_{(a,b,j)}\sigma_{(c,d,k)}\sigma_{(e,f,t)} | a, c, e \in A, \quad b, d, f \in B, \quad j, k, t \in I\} >$$

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THANKS FOR YOUR ATTENTION!