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Abstracts

A NEW FAMILY OF IRRETRACTABLE LEFT CYCLE SETS

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In this talk we introduce a new family of irretractable left cycle sets [2], which allow to construct irretractable square-free solutions of the Yang-Baxter equation that are new counterexamples to Gateva-Ivanova's Strong Conjecture [3]. In particular, this family includes some recent examples of Bachiller, Cedo, Jespers, Okninski [1].

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NORMALLY ζ-REVERSIBLE PROFINITE GROUPS

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Assume that G is a profinite group with the property that for each positive integer n, G contains only finitely many open normal subgroups of index n

For any $n \in \mathbb{N}$, let $a_n^{\triangleleft}(G)$ be the number of the open normal subgroups of G and let $b_n^{\triangleleft}(G) = \sum_{|G:H|=n, H\triangleleft_o G} \mu^{\triangleleft}(H,G)$, where $\mu^{\triangleleft}$ is the Möbius function in the lattice of the open normal subgroups of G.

The properties of the sequences $\{a_n^{\triangleleft}(G)\}_{n\in\mathbb{N}}$ and $\{b_n^{\triangleleft}(G)\}_{n\in\mathbb{N}}$ can be encoded by the corresponding Dirichlet generating function,

$$\zeta_G^{\triangleleft}(s) = \sum_{n\in\mathbb{N}} \frac{a_n^{\triangleleft}(G)}{n^s} \quad \text{and} \quad p_G^{\triangleleft}(s) = \sum_{n\in\mathbb{N}} \frac{b_n^{\triangleleft}(G)}{n^s}$$

called, respectively, the normal subgroup zeta function and the inverse of the normal probabilistic zeta function of G.

We will say that a profinite groups G is normally ζ-reversible if

$$\zeta_{G^{\triangleleft}}(s) p_G^{\triangleleft}(s) = 1,$$

and we conjecture that normally ζ-reversible profinite groups are pronilpotent.

We can prove this conjecture if G is a normally ζ-reversible satisying one of the following properties: G is prosoluble, G is perfect, all the nonabelian composition factors of G are alternating groups.

*This is a joint work with Andrea Lucchini.

INVOLUTION CONJUGACY CLASSES IN THE AFFINE COXETER GROUPS

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In general, Conjugacy classes have been powerful tools to study the structure of groups. In this talk, I shall discuss conjugacy classes of involutions for finite Coxeter groups and in particular affine Coxeter groups which is relevant in out further study of commuting involution graphs. For G a group and X is a subset of involutions of G. Then, the elements of X constitute the vertices of commuting graph which denoted $C(G; X)$. Where, $x, y \in X$ are joined by edge whenever $xy = yx$.

THE ALGEBRAIC STRUCTURE OF SEMI-BRACE

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Rump, in [4], introduced braces to study non-degenerate involutive solutions of the Yang-Baxter equation. Many aspects of this algebraic structure were studied and developed (see, for instance [2] and its bibliography). Recently, Guarnieri and Vendramin in [3] obtained a generalization of braces, skew braces, in order to construct non-degenerate bijective solutions of the Yang-Baxter equation.
In this talk, we focus on *semi-braces*, a further generalization of braces introduced in [1] that allows us to construct new solutions, not necessarily bijective. In particular, we describe the structural aspects of a semi-brace and provide a clear characterization of this structure. Finally, we introduce suitable concepts of ideal and quotient structure of a semi-brace.

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WEAKLY POWER AUTOMORPHISMS OF GROUPS

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An automorphism α of a group is said to be a *power automorphism* if it maps every subgroup of the group onto itself. The behaviour of power automorphisms and the structure of the group $PAut(G)$ of all power automorphisms of a group G have been studied by C. D. H. Cooper [1].

Here an automorphism α of a non-periodic group G will be called a *weakly power automorphism* if $\alpha(H) = H$ for every non-periodic subgroup H of G. Clearly, the set $WAut(G)$ of all weakly power automorphisms of G is a subgroup of the full automorphism group $Aut(G)$ and contains $PAut(G)$, so that it is natural to ask how much $PAut(G)$ is far from $WAut(G)$; some results concerning this problem will be presented, as a part of a discussion about the behaviour of weakly power automorphisms of non-periodic groups.

References

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GROUPS WITH PERMUTABILITY CONDITIONS ON SUBGROUPS OF INFINITE RANK

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A group G is said to have *finite (Prüfer) rank* r if every finitely generated subgroup of G can be generated by at most r elements, and r is the least positive integer with such a property; if such an r does not exist, we will say that the group G has *infinite rank*.

In this talk, generalized radical groups of infinite rank whose subgroups of infinite rank satisfy some generalized permutability conditions are considered.

REGULAR LANGUAGE EQUATIONS

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In this talk I will introduce the notion of regular language equations over a free group (or a free monoid). This is a wide generalization of the classical word equations over free groups and monoids, which have been the object of intense study at least for the past 50 years. The first breakthroughs on this topic were made by Makanin's proof of decidability (1977), and later by Razborov's description of sets of solutions (1987). These formed the ground base for the solutions to the Tarski problems given by Kharlampovich-Miasnikov and by Sela (2006).

After presenting and motivating the problem of regular language equations, I will address the question of whether these are decidable over a free group and over a free monoid. In the first case we use a generalization of Makanin-Razborov's process which relies, among others, on performing Stalling's foldings as an elementary transformation.

In a regular language equation over $F(A)$, one has a set X of variables, a set A of constants, and two finite regular automata $\mathcal{A}_1$, $\mathcal{A}_2$ on the alphabet $(X \cup A)^{\pm 1}$. A solution is an assignment of the variables $f : X \to F(A)$ so that, after replacing each $x \in X$ by $f(x)$, the languages accepted by $\mathcal{A}_1$ and $\mathcal{A}_2$ are the same in $F(A)$ (that is, up to reducing words).

FUSION AND PEARLS

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In finite group theory, the word fusion refers to the study of conjugacy maps between subgroups of a group. The modern way to solve problems involving fusion is via the theory of fusion systems. A saturated fusion system on a p-group S is a category whose objects are the subgroups of S and whose morphisms are the monomorphisms between subgroups which satisy certain axioms. To classify saturated fusion systems on S, we first need to determine the so-called essential subgroups of S. These are self-centralizing subgroups of S whose automorphism group has a restricted structure. If p is an odd prime, we call pearls the essential subgroups of S that are either elementary abelian of order p^2 or extraspecial of order p^3 and exponent p. In this talk we present new results about fusion systems involving pearls.

*This is a joint work with Alexei Miasnikov.

FINITE GROUPS AND LIE RINGS WITH A FROBENIUS GROUP OF AUTOMORPHISMS

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We recall that a finite *Frobenius group* FH, with kernel F and complement H, is a semidirect product of a normal subgroup F on which H acts by automorphisms so that each nontrivial element h in H is fixed point-free. Suppose that a finite group G admits FH as a group of automorphisms. If the subgroup $C_G(F)$ of the elements fixed by the kernel is trivial, then the group G has restricted structure: it is soluble and if the order of F is a prime number, then it is also nilpotent. Furthermore, the properties of G are expected to be close to the corresponding properties of $C_G(K)$, possibly depending also on the order of H. In this direction positive results have been obtained in [3] by E. Khukhro, P. Shumyatsky and N. Makarenko for the order of G, its rank, its Fitting height and its nilpotency class. In particular, they proved that the nilpotency of $C_G(H)$ implies the nilpotency of the group G. If the kernel F is cyclic, then the nilpotency class of G can be bounded from the above in terms of the nilpotency class of $C_G(H)$ and the order of H.

If F is not cyclic, then the triviality of $C_G(F)$ alone does not imply any bound for the nilpotency class and the derived lenght of G. However, we generalise the result to the case of an abelian non-cyclic kernel, under the additional hypothesis of H "being small" compared to F.

Theorem. *Let G be a finite group admitting a Frobenius group of automorphisms FH with abelian kernel F. Suppose that FH acts coprimely on G in such a manner that $C_G(F)$ is trivial and $C_G(H)$ is abelian. If $|H|^2$ is less than the least prime divisor of $|F|$, then G is nilpotent of class bounded in terms of the orders of F and H.*

The technique used to solve the nilpotency class problem is based on a *Lie ring method*. It was applied to the study of fixed-point free automorphisms of finite groups for the first time by G. Higman (see [1]). Whenever the group G is nilpotent we can construct the *associated Lie ring* $L(G)$ starting from its lower central series. The Lie ring $L(G)$ has nilpotency class equal to that of G. Moreover, the Frobenius group FH induces a Frobenius group of (Lie ring) automorphisms of $L(G)$. Because of this correspondence we can study the same problem for a Lie ring. Since its structure is more "regular" compared to the one of the groups, this translation will make the problem easier to solve. We use the decomposition in eigenspaces, which creates a grading of $L(G)$. The bound is then obtained by combinatorial tools.

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RELATIONAL COMPLEXITY FOR FINITE PERMUTATION GROUPS

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Relational complexity is an invariant of a finite permutation group and was introduced by Cherlin in 1996. It can be rather difficult to compute the relational complexity of any given permutation group. In this talk we will give a definition of relational complexity, we will consider some interesting examples, and we will give some ideas of how to calculate this invariant for some important classes of groups. We will focus particularly on primitive groups.

ZASSENHAUS CONJECTURE FOR SMALL GROUPS

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H. J. Zassenhaus conjectured in 1974 that any unit of finite order in the integral group ring $\mathbb{Z}G$ of a finite group G is as trivial as one can expect. More precisely, he conjectured that any unit of finite order is conjugate in the rational group algebra $\mathbb{Q}G$ to an element of the form $\pm g$ for some element $g \in G$. Although proven for several series of groups applying various methods, the conjecture remains open in general.

To determine the smallest possible counterexamples and analyse where we encounter complications in the proof of the Zassenhaus Conjecture we applied the available methods to all groups of order less than 288. I will sketch these methods and present ideas which might lead to a solution in the sixteen remaining cases in the range of groups.

REGULAR ORBITS OF FINITE SOLVABLE GROUPS

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All groups and modules in this talk will be finite.

The aim of the talk is to present an extension of some well-known results concerning regular orbits of solvable groups. Let G be a solvable group and V be a faithful and completely reducible G-module. Then G has at least three regular orbits on $V \oplus V$ provided that:

1. G is nilpotent (Moretó and Wolf, Adv.Math, 2004), or
2. G is a $3'$-group (Yang, Proc. Amer. Math. Soc.,2011), or

If the Sylow 2-subgroups of GV is abelian, then G has at least two regular orbits on $V \oplus V$ (Dolfi and Jabara, J. Algebra, 2007).

We take these studies very much further and show that if G is $SL(2,3)$-free and GV is S_4-free, then G has at least two regular orbits on $V \oplus V$; if, in addition, G is $\Gamma(2^3)$-free, G has at least three regular orbits on $V \oplus V$. As an application, we confirm Gluck's Conjecture about large character degrees for this class of groups.

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RIGHT ENGEL ELEMENTS IN THE FIRST GRIGORCHUK GROUP

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An element g of a given group G is a right Engel element if for every $x \in G$ there exists an integer $n = n(g,x) \geq 1$ such that $[g, x, \overset{n}{\ldots}, x] = 1$. Similarly, g is a left Engel element if the variable x appears on the left. The group G is said to be an Engel group if all its elements are right Engel or, equivalently, left Engel. It is a long-standing problem, raised by Plotkin, whether the sets of right Engel elements and left Engel elements of a given group G are subgroups. In 2005, Bludov announced that there exists a group in which the set of left Engel elements is not a subgroup. This example is based on the first Grigorchuk group and it has been refined by Bartholdi who proved that the first Grigorchuk group itself is not Engel. In this talk, we first introduce the Grigorchuk group and then we show that the only right Engel element in the first Grigorchuk group is the identity.

*This is a joint work with Adolfo Ballester-Bolinches.

AN ELEMENTARY PROOF OF GRAHAM'S THEOREM

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Semigroup theory has been of particular importance in theoretical computer science since the 1950s because of the natural link between finite semigroups and finite automata. First introduced and studied by Rees in his 1940's seminar papaer, 0-simple semigroups are considered one of the keystones in semigroup theory playing a significant role in its development. In fact, a detailed study of the idempotent-generated subsemigroup of a given semigroup turns out to be crucial for understanding complexity, which is one of the most famous problems in semigroup theory. In this context, Graham's theorem has become one of the most important basic results. Although the original and alternative proofs involve graph theory or topological techniques and cohomology, we present a very elementary proof applying basic results on regularity to the description of 0-simple semigroups given by Rees.

A NEW CONSTRUCTION OF INDECOMPOSABLE SOLUTIONS OF THE YANG-BAXTER EQUATION

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In 1992, Drinfeld [2] posed the question of finding all set-theoretic solutions of the Yang-Baxter equation. Rump [4] defined new algebraic structure, the cycle sets and proved that there is a bijective correspondence between a subfamily of these solutions and the left cycle sets. In order to find new solutions of the Yang-Baxter equation we introduce a new contruction of left cycle sets [1]. Moreover we give a new method to contruct, in particular, indecomposable cycle sets [**?**] and hence new indecomposable solutions [3].

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HIERARCHICALLY HYPERBOLIC GROUPS, CUBULATING GROUPS AND RAAGS

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Hierarchically Hyperbolic Groups is a notion recently introduced by Behrstock, Hagen and Sisto in a series of papers [1] and [2]. These groups are far-reaching generalizations of hyperbolic groups and hyperbolic spaces, mapping class groups, Teichmüller spaces, cubulable groups and right-angled Artin groups. In this talk we will see the main definition for HHGroups and HHSpaces, go through some of the motivation and examples and talk about some handy basic consequences to get a feeling of what these particular type of groups look like. If the time allows it, we will have a look at some of the questions currently being worked on and I will also present a recent result on combination theorem for graph of hierarchically hyperbolic groups which is a slightly strengthened version to that presented in [2].

References

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ON THE ZASSENHAUS CONJECTURE FOR DIRECT PRODUCTS

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H.J. Zassenhaus conjectured that any unit of finite order and augmentation one in the integral group ring ZG of a finite group G is conjugate in the rational group algebra QG to an element of G. The Zassenhaus Conjecture found much attention and was proved for many series of groups, e.g. for nilpotent groups, groups possessing a normal Sylow subgroup with abelian complement or cyclic-by-abelian groups. However, there is no so much information about the conjecture for direct products. In this talk we present our recent results on the Zassenhaus Conjecture for the direct product $G \times A$, where G is a Camina finite group and A is an abelian finite group.

SEMI-BRACES AND THE YANG-BAXTER EQUATION

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The Yang-Baxter equation is a basic equation of the statistical mechanics that arose from Yang's work in 1967 and Baxter's one in 1972.

Drinfeld in [2] posed the question of classifying the solutions of the Yang-Baxter equation, in particular those called set-theoretical. This is a difficult task and many authors dealt with this problem.

In particular, several algebraic structures were studied to answer this problem, such as groups, cycle sets, braces (for instance, see [3], [4], [5]).

Recently, a new generalization of braces, *semi-brace*, was introduced in [1].

In this talk, we describe how to obtain a solution of the Yang-Baxter equation through semi-braces. Furthermore, we show which properties satisfy this kind of solutions.

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A STILL UNDECIDED POINT ON GROUPS WITH AN IDENTITY

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The origin of the general Burnside problem is the famous paper [2] where W. Burnside posed his attention to "a still undecided point" on periodic groups. More precisely, the question was whether a finitely generated periodic group is necessarily finite. Burnside immediately suggested the "easier" question about the finiteness of a finitely generated periodic group of *bounded exponent*. After some partial results, this latter was replaced with the so-called restricted Burnside problem: given two positive integers d and n, is there a largest finite d-generator group of exponent n? Unlike the first two questions, the restricted Burnside problem has a positive answer. His solution is due to Zelmanov and the proof is based on some deep results on Engel Lie algebras (see [4] for an account).

Let $w = w(x_1, \ldots, x_m)$ be a nonempty word in the free group generated by $x_1, \ldots, x_m$. A group G is said to satisfy the identity $w \equiv 1$ if $w(g_1, \ldots, g_m) = 1$ for all $g_1, \ldots, g_m \in G$. As a consequence of the positive solution of the restricted Burnside problem, if $w = x^n$ for some $n \geq 1$, and G is a finitely generated residually finite group, then G is finite: namely, every residually finite group of finite exponent is locally finite. In this context, Zelmanov has recently proved that a residually finite p-group, for p a prime, which satisfies an arbitrary identity is locally finite [5]. The result was already announced in [4] where it was also conjectured the same for periodic groups. However, it is still unclear how to deal with the periodic case. In this talk, as the Burnside problems are closely related to the theory of Engel groups (see, for instance, [3]), we will discuss an analogous result and some consequences for groups generated by bounded Engel elements [1].

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COLEMAN AUTOMORPHISMS OF FINITE GROUPS AND THEIR MINIMAL NORMAL SUBGROUPS

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We quickly cover the basics of Coleman automorphisms of finite groups and why they are relevant to the well known normalizer problem. The bulk of the talk will be on our partial answers to the questions on Coleman automorphisms posed by Hertweck and Kimmerle in their well-known paper "Coleman automorphisms of finite groups". As an apotheosis, we will mention several open questions for further work in this area of research.

GROUPS IN WHICH EVERY PROPER SUBGROUP OF INFINITE RANK IS FINITE RANK-BY-HYPERCENTRAL OR HYPERCENTRAL-BY-FINITE RANK

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Let $\mathcal{P}$ be a group theoretical property or a class of groups. One of the main topics of infinite group theory is to study strongly locally graded groups of infinite rank G whose proper subgroups of infinite rank belong to $\mathcal{P}$. Thus, the aim is to understand how the group G can be influenced by this condition.

*This is a joint work with Nadir Trabelsi.

Are all proper subgroups of G in $\mathcal{P}$? Does G itself belong to $\mathcal{P}$? Many authors studied groups of infinite rank with different choices of the property $\mathcal{P}$ (see for instance [1],[2],[3],[4]). The types of classes $\mathcal{P}$ that we are interested in are: $\mathcal{N}\mathcal{X}$ (resp. $\mathcal{N}_c\mathcal{X}$,$\mathcal{Z}\mathcal{A}\mathcal{C}$), and their duals $\mathcal{X}\mathcal{N}$ (resp. $\mathcal{X}\mathcal{N}_c$,$\mathcal{C}\mathcal{Z}\mathcal{A}$), where $\mathcal{N}, \mathcal{N}_c, \mathcal{Z}\mathcal{A}, \mathcal{C}$, are respectively, the class of nilpotent, nilpotent of class c ($c \geq 1$ an integer), hypercentral, Černikov, and $\mathcal{X}$ is any class contained in the class of finite rank groups.

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