New variations on the theme of Baer's theorem

Liliana Lancellotti

Joint work with **M. Capasso** and **P. Shumyatsky**



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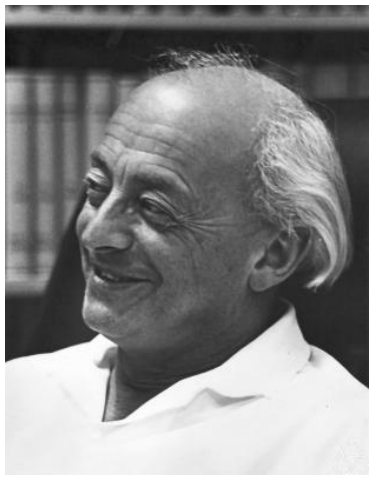


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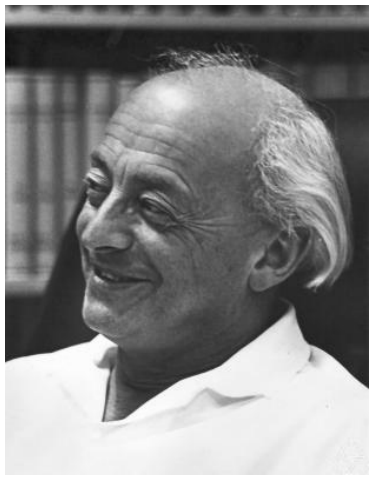
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Let $t \geq 1$. If $G/Z_t(G)$ is finite, then $\gamma_{t+1}(G)$ is finite.

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I. Schur (case $t = 1$) (1904)

If $G/Z(G)$ is finite, then G' is finite.

J. Wiegold (1965)

If $|G/Z(G)| = n < \infty$, *then* $|G'| \leq \omega(n) = n^{\frac{\log_2 n - 1}{2}}$.

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L. A. Kurdachenko and I. Ya. Subbotin (2013)

Let $t \geq 1$. *If* $|G/Z_t(G)| = n < \infty$, *then* $|\gamma_{t+1}(G)| \leq \beta(n,t)$ *where* $\beta(n,1) = \omega(n)$ *and* $\beta(n,t) = \omega(\beta(n,t-1)) \cdot \beta(n,t-1)^n$.

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We use the expression *"*$(a, b, \dots)$*-bounded"* to mean that a quantity is finite and bounded by a certain number depending only on the parameters $a, b, \dots$.

Question

Find those natural classes of groups $\mathfrak{X}$ for which analogues of Baer's theorem remain valid for every group G and $t \geq 1$.

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2. obtain bounds depending only on the initial parameters.

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$\mathfrak{X}$	*Baer*
Finite	✓

L.A. Kurdachenko and P. Shumyatsky (2013)

There exists a group G having the following properties:

(i) $G = G'$;

(ii) $G/Z(G)$ *is a* **Tarski group**, *i.e. an infinite p-group, whose proper non-trivial subgroups have order p, where p is a prime such that $p > 10^{75}$;*

(iii) $Z(G)$ *is a free abelian group of infinite torsion-free rank.*

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(iii) $Z(G)$ *is a free abelian group of infinite torsion-free rank.*

The factor group $G/Z(G)$ is a **Tarski p-group**, so that it is a periodic group, having finite exponent. It also satisfies the maximal and minimal conditions on subgroups, has Prüfer rank 2 and sectional p-rank 1.

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$\mathfrak{X}$	*Baer*
Finite	✓
Periodic	Schur: ✗
Finite exponent	Schur: ✗
Finite Prüfer rank	Schur: ✗
Finite sectional p-rank	Schur: ✗
Min	Schur: ✗
Max	Schur: ✗

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A group G has *finite (Prüfer) rank* if there is a positive integer r such that every finitely generated subgroup of G can be generated by at most r elements. The least such r is the *rank of* G.

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Finite	✓
Finite Prüfer rank + G finite	✓

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Finite	✓
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A group G has *finite (Prüfer) rank* if there is a positive integer r such that every finitely generated subgroup of G can be generated by at most r elements. The least such r is the *rank of* G.

A group G is *generalized radical* if it has an ascending series whose factors are locally nilpotent or locally finite. It is *locally generalized radical* if every finitely generated subgroup is generalized radical.

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Let p be a prime. A group G has *finite sectional p-rank* if there exists a non-negative integer r such that every elementary abelian p-section of G has order at most p^r. The least such r is called the *sectional p-rank of G*.

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Finite	✓
Finite Prüfer rank + G locally generalized radical	✓
Finite sectional p-rank + G locally generalized radical	✓

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- $m = m(G) \geq 0$ and $n = n(G) \geq 1$ are invariants of G and the pair (m, n) is called the *size of* G.
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Finite	✓
Finite Prüfer rank + G locally generalized radical	✓
Finite sectional p-rank + G locally generalized radical	✓
Chernikov	✓
Polycyclic-by-finite	✓

- By Baer's theorem, for $t \geq 1$,

$$\left| \frac{G}{Z_t(G)} \right| = n < \infty \implies |\gamma_{t+1}(G)| \text{ is } (n,t)\text{-bounded.}$$

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- The general case follows by induction on t.

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  of (n, s)-bounded order.
- Now apply induction on t to the pair $(s + 1, t - 1)$ to obtain that $\gamma_{s+t}(G)$ has (n, s, t)-bounded order.



G. Fernández-Alcober and M. Morigi (2009).
Generalizing a theorem of P. Hall on finite-by-nilpotent groups.
Proceedings of the American Mathematical Society, **137**(2), 425–429.



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Let $s \geq 1$. If G is a group such that $\gamma_s(G)/(\gamma_s(G) \cap Z(G))$ is finite of order n, then $\gamma_{s+1}(G)$ is finite of (n, s)-bounded order.



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Let $s, t \geq 1$. If G is a group such that $\gamma_s(G)/(\gamma_s(G) \cap Z_t(G))$ is finite of order n, then $\gamma_{s+j}(G)/(\gamma_{s+j}(G) \cap Z_{t-j}(G))$ is finite of (n, s, j)-bounded order for every j with $0 \leq j \leq t$.
In particular $\gamma_{s+t}(G)$ is finite of (n, s, t)-bounded order.

Question

Find those natural classes of groups $\mathfrak{X}$ for which analogues of Fernández-Alcober and Morigi's theorem remain valid for every group G and $s, t \geq 1$.

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$\mathfrak{X}$	*Baer*	*Alcober–Morigi* $(t = 1)$	*Alcober–Morigi*
Finite	✓	✓	✓

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$\mathfrak{X}$	*Baer*	*Alcober–Morigi* $(t = 1)$	*Alcober–Morigi*
Finite	✓	✓	✓
Finite Prüfer rank + G locally generalized radical	✓		
Finite sectional p-rank + G locally generalized radical	✓		
Chernikov	✓		
Polycyclic-by-finite	✓		

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Finite	✓	✓	✓
Finite Prüfer rank + G locally generalized radical	✓		
Finite sectional p-rank + G locally generalized radical	✓		
Chernikov	✓		
Polycyclic-by-finite	✓	✓	✓

1. $\gamma_{\boldsymbol{s}}(G)/(\gamma_{\boldsymbol{s}}(G) \cap Z_{\boldsymbol{t}}(G)) \in \mathfrak{X} \implies \gamma_{\boldsymbol{s}+\boldsymbol{t}}(G) \in \mathfrak{X}$;
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$\mathfrak{X}$	*Baer*	*Alcober–Morigi* $(t = 1)$	*Alcober–Morigi*
Finite	✓	✓	✓
Finite Prüfer rank + G finite nilpotent	✓	✓	✓
Finite sectional p-rank + G locally generalized radical	✓		
Chernikov	✓		
Polycyclic-by-finite	✓	✓	✓

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$\mathfrak{X}$	*Baer*	*Alcober–Morigi* $(t = 1)$	*Alcober–Morigi*
Finite	✓	✓	✓
Finite Prüfer rank + G finite	✓	✓	✓
Finite sectional p-rank + G locally generalized radical	✓		
Chernikov	✓		
Polycyclic-by-finite	✓	✓	✓

M. Capasso, L. L. and P. Shumyatsky (case $t = 1$) (2026)

Let $s \geq 1$ and let G be a finite group. If $\gamma_s(G)/(\gamma_s(G) \cap Z(G))$ has rank r, then $\gamma_{s+1}(G)$ has (r, s)-bounded rank.

Proof idea.

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- By Lubotzky–Mann (1987), we may assume that $\gamma_s(G)$ is abelian.
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Proof idea.

- By Lubotzky–Mann (1987), we may assume that $\gamma_s(G)$ is abelian.
- Jamali–Zandi (2013) bounds the rank of $\gamma_{s+1}(G)/\gamma_\infty(G)$.
- Coprime action embeds each Sylow subgroup of $\gamma_\infty(G)$ into $\gamma_s(G)/(\gamma_s(G) \cap Z(G))$, bounding the rank of $\gamma_\infty(G)$.

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Chernikov	✓		
Polycyclic-by-finite	✓	✓	✓

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Polycyclic-by-finite	✓	✓	✓

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Finite sectional p-rank + G locally generalized radical	✓	✓	✓
Chernikov	✓		
Polycyclic-by-finite	✓	✓	✓

M. Capasso, L. L. and P. Shumyatsky (case $t = 1$) (2026)

Let $s \geq 1$, let p be a prime and let G be a locally generalized radical group. If $\gamma_s(G)/(\gamma_s(G) \cap Z(G))$ has sectional p-rank r, then $\gamma_{s+1}(G)$ has finite (r, s)-bounded sectional p-rank.

Proof idea.

M. Capasso, L. L. and P. Shumyatsky (case $t = 1$) (2026)

Let $s \geq 1$, let p be a prime and let G be a locally generalized radical group. If $\gamma_s(G)/(\gamma_s(G) \cap Z(G))$ has sectional p-rank r, then $\gamma_{s+1}(G)$ has finite (r, s)-bounded sectional p-rank.

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- We may assume that $\gamma_s(G)$ is abelian (Ballester-Bolinches et al. (2013)) and that $\gamma_s(G)$ has no non-trivial elements of finite order coprime to p.

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- Our rank bound controls the Prüfer rank of $\gamma_{s+1}(G)$ and so also its sectional p-rank.

1. $\gamma_{s}(G)/(\gamma_{s}(G) \cap Z_{t}(G)) \in \mathfrak{X} \implies \gamma_{s+t}(G) \in \mathfrak{X}$;
2. obtain bounds depending only on the initial parameters.

$\mathfrak{X}$	*Baer*	*Alcober–Morigi* ($t = 1$)	*Alcober–Morigi*
Finite	✓	✓	✓
Finite Prüfer rank + G locally generalized radical	✓	✓	✓
Finite sectional p-rank + G locally generalized radical	✓	✓	✓
Chernikov	✓		
Polycyclic-by-finite	✓	✓	✓

M. Capasso, L. L. and P. Shumyatsky (case $t = 1$) (2026)

Let $m \geq 0$, $n, s \geq 1$ and let G be a group. If $\gamma_s(G)/(\gamma_s(G) \cap Z(G))$ is Chernikov of size (m, n), then $\gamma_{s+1}(G)$ is Chernikov of (m, n, s)-bounded size.

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- Then $\gamma_{s+1}(G)$ is periodic, with at most $m + n$ prime divisors, so it suffices to assume that $\gamma_{s+1}(G)$ is a p-group.

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- Our rank bound controls the number of Prüfer factors in the radicable part D of $\gamma_{s+1}(G)$.
- Applying Fernández-Alcober–Morigi (2009) to G/D bounds the order of $\gamma_{s+1}(G)/D$.

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M. Capasso, L. Lancellotti and P. Shumyatsky (2026).
New variations on the theme of Baer's theorem.
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Thank you