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Torsion Locally Nilpotent Groups with the non-Dedekind Norm of Decomposable Subgroups

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Abstract

We study torsion locally nilpotent groups with the non-Dedekind norm of decomposable subgroups. It is found out that such groups are locally finite p-groups. Their structure is described.

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1 Introduction

In general group theory, a large number of findings relates to the study of groups, in which subgroups or their systems have some properties. In some cases the presence of even one (and, as a rule, characteristic) subgroup with a given property can significantly impact the structure of a group. Different Σ-norms of a group are subgroups of such a type.

Let Σ be the system of all subgroups of a group G that have some theoretical group property. The Σ-*norm* of a group G is the intersection $N_\Sigma(G)$ of the normalizers of all subgroups of the system Σ. By definition, the Σ-norm is a characteristic subgroup and contains the center. Moreover, the Σ-norm is the maximal subgroup of a group that normalizes all Σ-subgroups of that group (if $\Sigma \neq \emptyset$). Therefore, all subgroups of the Σ-norm that belong to the system Σ are normal in $N_\Sigma(G)$ (although there may not be such subgroups).

When considering Σ-norms, a number of questions related to the study of properties of a group with the given system Σ of subgroups and some restrictions on the Σ-norm arise. Specific problems of this kind were solved by many researchers depending on the choice of the system Σ and properties of the Σ-norm (see for instance [1]–[5], [7]–[12], [20], [21]).

In many cases, the structure of the Σ-norm and the nature of its embedding in a group give the opportunity to characterize the properties of a group. For example, in a group which is finite extension of the Σ-norm, the normalizers of all subgroups of the system Σ are of finite (in particular, identity) index. Moreover, all subgroups from the system Σ are normal in a group which coincides with its Σ-norm.

Groups with different systems Σ of normal subgroups were actively studied in the second half of the XX century. Therefore, for many systems Σ of subgroups the structure of groups, which coincide with their Σ-norm, is well-known. So, the question about the properties of groups in which the Σ-norm satisfies some restrictions and is a proper subgroup of a group is natural.

For the first time, such a problem was considered by R. Baer in the 30s of the XX century [1] for the system Σ, consisting of all subgroups of a group. R. Baer called this Σ-norm the norm $N(G)$ of a group G. Later, R. Baer's idea of defining the norm of a group was transferred to other systems Σ of subgroups (see for instance [2, 4, 5, 10, 16, 20, 21]). It is clear that the norm $N(G)$ is contained in all other Σ-norms, which can be considered as its generalizations.

The authors consider one of such generalizations, i.e. the *norm of decomposable subgroups* of a group. According to [15], it is the intersection N_G^d of normalizers of all decomposable subgroups of a group or a group itself, if the system of such subgroups is empty. A subgroup of G is *decomposable* if it can be represented as the direct product of two of non-trivial factors [13].

By the definition of the norm N_G^d we get that in the case $N_G^d = G$ decomposable subgroups of a group G are either normal, or the set of such subgroups is empty. Non-Abelian groups with this property were considered in [13] and were called di*-groups*. Therefore, the structure of groups in which this norm coincides with a group is known. The authors continue the study of groups with the non-Dedekind norm of decomposable subgroups, started in [8], [9], [15]–[17].

Since the condition of the existence of decomposable subgroups in a group is equivalent to the existence of decomposable Abelian subgroups in it, the norm of decomposable subgroups was studied depending on the presence of systems of decomposable Abelian subgroups in a group. This fact also gives reason to believe that the norm N_G^d of decomposable subgroups is somehow related to the norm N_G^A of Abelian non-cyclic subgroups of a group (see [7]). The norm N_G^A is the intersection of the normalizers of all Abelian non-cyclic subgroups of a group, provided that the system of such subgroups in a group is not empty.

This agreed with the results of [15], where it was established that in the class of locally finite p-groups these norms coincide (Theorem 1.1). So, under the condition of existence of at least one Abelian non-cyclic subgroup in such a group, the norm N_G^d of decomposable subgroups of G is non-Dedekind if and only if the norm N_G^A of Abelian non-cyclic subgroups is non-Dedekind. The last remark allows us to use the results of findings [3, 7, 12, 14, 18, 19] and to characterize locally finite p-groups in which the norm of decomposable subgroups is non-Dedekind.

In the classes of finite non-primary and infinite torsion locally nilpotent non-primary groups, these norms are interconnected by the inclusion $N_G^A \supseteq N_G^d$, moreover there are groups in which the norm N_G^d is a proper subgroup of the norm N_G^A (see [15], Theorems 1.2 and 1.3).

The purpose of this article is to study the properties of torsion locally nilpotent groups, in which the norm of decomposable subgroups is non-Dedekind.

2 Preliminary results

As mentioned above, groups which coincide with the norm N_G^d of decomposable subgroups belong to the class of di-groups. The structure of torsion locally nilpotent di-groups is characterized by the following statement.

Proposition 1 (see [13]) *Any torsion locally nilpotent non-Hamiltonian* di*-group is a* p*-group of one of the following types:*

1) *a quaternion 2-group of order greater then 8 (finite or infinite);*

2) *a locally finite non-Hamiltonian* p*-group, in which the set of decomposable subgroups is non-empty, and every Abelian non-cyclic subgroup is normal.*

The first type of a group in Proposition 1 characterizes the structure of torsion locally nilpotent di-groups which do not contain decomposable subgroups, and the second one characterizes di-groups in which the system of decomposable subgroups is non-empty and consists of exactly normal subgroups. In the last case di-groups are locally finite p-groups, whose all Abelian non-cyclic subgroups are normal. Non-Abelian groups with this property are called $\overline{HA}$*-groups* ($\overline{HA}_p$*-groups*, if they are p-groups) and were fully described by F.M. Lyman in [6].

Thus, an arbitrary torsion locally nilpotent di-group G with Abelian non-cyclic subgroup is a $\overline{HA}_p$-group and $G = N_G^d = N_G^A$. So, the structure of torsion locally nilpotent groups that coincide with the norm of decomposable subgroups is known. Let's show that in the class of torsion locally nilpotent groups a rather similar situation holds in the general case, provided that the norm of decomposable subgroups is non-Dedekind.

Let's formulate some evident properties of the norm N_G^d of decomposable subgroups of a group G, which follow from its definition.

Lemma 2 *Let G be a group and N_G^d be the norm of decomposable subgroups. Then the following statements hold:*

1) $N_G^d \supseteq N(G) \supseteq Z(G)$, *where $Z(G)$ is the center and $N(G)$ is the norm of a group G;*

2) $N_G^d = N^d_{N_G^d}$;

3) *any decomposable subgroup of the norm N_G^d is normal in it;*

4) *if H is a subgroup of a group G, then $H \bigcap N_G^d \subseteq N_H^d$;*

5) *if $N_G^d \subseteq H$, then $N_G^d \subseteq N_H^d$;*

6) *if H is an Abelian subgroup and $H \subseteq C_G(N_G^d)$, then the group $G_1 = H \cdot N_G^d$ is either Dedekind or non-Hamiltonian* di*-group and $G_1 = N^d_{G_1}$ in both cases.*

The following statement will be actively used later.

Proposition 3 (see [16], Theorem 1.4) *Any group* G *with non-Dedekind norm* N_G^d *of decomposable subgroups contains non-primary Abelian subgroups if and only if its norm* N_G^d *contains subgroups with such properties.*

3 Main results

Let's consider torsion locally nilpotent groups with the non-Dedekind norm N_G^d of decomposable subgroups. Since N_G^d is a locally nilpotent non-Hamiltonian di-group in this case, by the description of such groups (Proposition 1) we get the following statement.

Lemma 4 *The norm* N_G^d *of decomposable subgroups of a torsion group* G *is non-Dedekind and locally nilpotent if and only if* N_G^d *is a quaternion 2-group of order greater than 8 (finite or infinite) or a non-Hamiltonian* $\overline{HA}_p$*-group.*

Corollary 5 *If the norm* N_G^d *of decomposable subgroups of a locally finite group* G *is a non-primary locally nilpotent subgroup, then it is Dedekind.*

Let's present some properties of torsion groups with the locally nilpotent non-Dedekind norm N_G^d.

Lemma 6 *If the norm* N_G^d *of decomposable subgroups of a torsion group* G *is non-Dedekind and locally nilpotent, then a group* G *does not contain non-primary cyclic subgroups.*

Proof — Let a group G and its norm N_G^d satisfy the conditions of the lemma. Then, by Lemma 4, N_G^d is a p-group and does not contain non-primary Abelian (in particular, cyclic) subgroups. By Proposition 3 a group G does not contain such subgroups. The lemma is proved. □

Corollary 7 *Any torsion group* G *with the non-Dedekind locally nilpotent norm* N_G^d *of decomposable subgroups and the non-identity center* $Z(G)$ *is a* p*-group.*

Proof — Let G be a torsion group with the locally nilpotent norm N_G^d of decomposable subgroups. By Lemma 6 a group G does not contain non-primary cyclic subgroups. Thus, if $Z(G) \neq E$, then G is a p-group. □

Lemma 8 *If the norm N_G^d of decomposable subgroups of a torsion locally nilpotent group G is non-Dedekind, then G is a locally finite p-group.*

Proof — As it is known, a torsion locally nilpotent group is the direct product of its Sylow p-subgroups. Since G does not contain non-primary cyclic subgroups by Lemma 6, then G is a p-group. □

By Lemma 8 the study of torsion locally nilpotent groups with the non-Dedekind norm N_G^d is reduced to the study of locally finite p-groups with the same restriction on the norm N_G^d.

The properties of such groups are fully characterized by the following Theorem 9 and Theorem 11. Let's note that Theorem 9 generalizes Lemma 2.1 of [15] for torsion locally nilpotent groups.

Theorem 9 *The norm N_G^d of a torsion locally nilpotent group G is non-Dedekind and does not contain decomposable subgroups if and only if $G = N_G^d$ and G is a quaternion 2-group of order greater than 8 (finite or infinite).*

Proof — The sufficiency of the conditions of the lemma follows from Proposition 1. Let's prove their necessity.

Let G be a torsion locally nilpotent group and its norm N_G^d be non-Dedekind. Then, by Lemma 8, G is a locally finite p-group. Since N_G^d does not contain decomposable subgroups, then $p = 2$ and N_G^d is a quaternion 2-group (finite or infinite) by Lemma 4. Moreover

$$N_G^d = A\langle b\rangle,$$

where $b^2 \in A$, $|b| = 4$, A is a cyclic or quasicyclic 2-group, $|A| > 4$ and $b^{-1}ab = a^{-1}$ for an arbitrary element $a \in A$.

Let us show that G contains one involution. Suppose contrary and $x \in G\backslash N_G^d$, $|x| = 2$. Then $[x, b^2] = 1$, where b^2 is the involution of the norm N_G^d. Since $\langle x, b^2\rangle$ is N_G^d-admissible, then

$$\langle x, b^2\rangle \lhd G_1 = \langle x\rangle N_G^d$$

and $\left[G_1 : C_{G_1}\left(\langle x, b^2\rangle\right)\right] \leqslant 2$.

If $[x, b] \neq 1$, then $[x, b] = b^2$ and $|xb| = 2$. Then the subgroup $\langle xb, b^2\rangle$ is Abelian, and, consequently, N_G^d-admissible, which is impossible because the element $a \in A$, $|a| = 8$ does not belong to the normalizer $N_G\left(\langle xb, b^2\rangle\right)$ of this subgroup. Therefore, $[x, b] = 1$. Since $\langle x, b\rangle$ is decomposable Abelian, it is N_G^d-admissible. But even in this case, the element $a \in A$, $|a| = 8$ does not belong to the normalizer of the subgroup $\langle x, b\rangle$.

Thus, G contains only one involution, and all its Abelian subgroups are indecomposable. By Proposition 1, G is a quaternion 2-group (finite or infinite). Since the norm N_G^d is non-Dedekind by the condition of the theorem, $|G| > 8$ and $G = N_G^d$. The theorem is proved. □

Corollary 10 *A torsion locally nilpotent group* G *with the non-Dedekind norm* N_G^d *does not contain decomposable subgroups if and only if the norm* N_G^d *does not contain such subgroups.*

Taking into account Theorem 9, it is easy to prove that in an infinite torsion locally nilpotent group G, which does not contain decomposable subgroups and has the non-Dedekind norm N_G^d, all Abelian non-cyclic subgroups are normal and $N_G^A = N_G^d$.

Let's consider torsion locally nilpotent groups with the non-Dedekind norm N_G^d provided that a group contains a decomposable (decomposable Abelian) subgroup. Their properties are characterized by Theorem 11, which actually reduces the study of such groups to the study of locally finite p-groups with the non-Dedekind norm N_G^A.

Theorem 11 *A torsion locally nilpotent group* G *with a decomposable Abelian subgroup has the non-Dedekind norm* N_G^d *of decomposable subgroups if and only if* G *is a locally finite* p*-group with the non-Dedekind norm* N_G^A *of Abelian non-cyclic subgroups and* $N_G^A = N_G^d$.

Proof — The sufficiency of the conditions of the theorem follows from Theorem 1.1 of [15]. Let's prove their necessity. Let G be a torsion locally nilpotent group with the non-Dedekind norm N_G^d of decomposable subgroups. Then by Lemma 8 G is a locally finite p-group for some prime p. Since a group contains decomposable subgroups by the condition of the theorem, it contains decomposable Abelian subgroups which are non-cyclic in the class of p-groups. By Theorem 1.1 of [15] in the class of locally finite p-groups

$$N_G^A = N_G^d.$$

So, G is a p-group with the non-Dedekind norm N_G^A of Abelian non-cyclic subgroups. The theorem is proved. □

Let's note that locally finite p-groups with the non-Dedekind norm N_G^A of Abelian non-cyclic subgroups were studied in detail by the authors earlier (see [3, 7, 12, 14, 18, 19]). So, the statements below are actually corollaries from Theorem 9, Theorem 11 and the results of above-mentioned findings.

In particular, by the description of infinite locally finite p-groups which contain Abelian non-cyclic subgroups and have the non-Dedekind norm N_G^A [7, 14], we get the following result, which fully describes the structure of infinite torsion locally nilpotent groups with the non-Dedekind norm N_G^d.

Theorem 12 *An infinite torsion locally nilpotent group G has the non-Dedekind norm N_G^d if and only if it is a p-group of one of the following types:*

1) $G = A\langle b\rangle$, *where A is a quasicyclic 2-group,* $|b| = 4$, $b^2 \in A$, $b^{-1}ab = a^{-1}$ *for any element* $a \in A$, $N_G^d = G$;

2) $G = A\langle b\rangle$, *where A is a quasicyclic 2-group,* $|b| = 8$, $b^4 \in A$, $b^{-1}ab = a^{-1}$ *for any element* $a \in A$, $N_G^d = G$;

3) $G = (A \times \langle b\rangle) \ltimes \langle c\rangle$, *where A is a quasicyclic p-group,* $|b| = |c| = p$, $[A, \langle c\rangle] = E$, $[b, c] = a_1 \in A$, $|a_1| = p$; $N_G^d = G$;

4) $G = A \times Q$, *where A is a quasicyclic 2-group, Q is the quaternion group of order 8,* $N_G^d = G$;

5) $G = \big(A \times \langle b\rangle\big) \ltimes \langle c\rangle \ltimes \langle d\rangle$, *where A is a quasicyclic 2-group,* $|b| = |c| = |d| = 2$, $[A, \langle c\rangle] = E$, $[b, c] = [b, d] = [c, d] = a_1 \in A$, $|a_1| = 2$, $d^{-1}ad = a^{-1}$ *for all* $a \in A$; $N_G^d = \big(\langle a_2\rangle \times \langle b\rangle\big) \ltimes \langle c\rangle$, $a_2 \in A$, $|a_2| = 4$;

6) $G = \big(A\langle y\rangle\big)Q$, *where A is a quasicyclic 2-group,* $[A, Q] = E$, $Q = \langle q_1, q_2\rangle$, $|q_1| = 4$, $q_1^2 = q_2^2 = [q_1, q_2]$, $|y| = 4$, $y^2 = a_1 \in A$, $|a_1| = 2$, $y^{-1}ay = a^{-1}$ *for all* $a \in A$, $\big[\langle y\rangle, Q\big] \subseteq \big\langle a_1, q_1^2\big\rangle$; $N_G^d = \langle a_2\rangle \times Q$, $a_2 \in A$, $|a_2| = 4$.

Proof — The sufficiency of the conditions of the theorem is easy to verify directly, so it remains to prove only the necessity.

Let a group G and it's norm N_G^d satisfy the condition of the theorem. If G does not contain decomposable subgroups, then, by Theorem 9, $G = N_G^d$ and G is an infinite quaternion 2-group, i.e. a group of the type 1) of this theorem. Let G contain decomposable (therefore, decomposable Abelian) subgroups. Then, by Theorem 11, G is a locally finite p-group with the non-Dedekind norm N_G^A of Abelian non-cyclic subgroups and $N_G^d = N_G^A$. By the description of infinite locally finite p-groups with the non-Dedekind norm N_G^A of Abelian non-cyclic subgroups (see [7],[14]), G is a group of one of the types 2)–6) of this theorem. □

Thus, the class of infinite torsion locally nilpotent groups with the non-Dedekind norm of decomposable subgroups coincides with the class of infinite locally finite p-groups with the same restriction on the norms of decomposable and Abelian non-cyclic subgroups.

The following corollaries follow directly from Theorem 12.

Corollary 13 *Any infinite torsion locally nilpotent group G with the non-Dedekind norm N_G^d is a finite extension of a quasicyclic p-subgroup.*

Corollary 14 *If the norm N_G^d of a torsion locally nilpotent group G is infinite and non-Dedekind, then all Abelian non-cyclic and all decomposable subgroups are normal in G.*

Corollary 15 *The norm N_G^d of decomposable subgroups of an infinite torsion locally nilpotent group G is Dedekind, if $1 < \left[G : N_G^d\right] < \infty$.*

Corollary 16 *Any infinite torsion locally nilpotent central-by-finite group G with the non-Dedekind norm N_G^d of decomposable subgroups is a p-group and $G = N_G^d$.*

Corollary 17 *If an infinite torsion locally nilpotent group G has the non-Dedekind norm N_G^d of decomposable subgroups and $2 \notin \pi(G)$, then G is a finite extension of the center $Z(G)$.*

Corollary 18 *Any infinite torsion locally nilpotent group G with the non-Dedekind norm N_G^d has non-identity center.*

Let's consider finite nilpotent groups with the non-Dedekind norm of decomposable subgroups. Their detailed description is in the following Theorem 19.

Theorem 19 *Any finite nilpotent group G with the non-Dedekind norm N_G^d of decomposable subgroups is a p-group of one of the following types:*

1) G *is a* $\overline{HA}_p$*-group;* $G = N_G^d$ *(p is prime);*

2) $G = \langle a \rangle \langle b \rangle$, *where* $|a| = 2^n$, $n > 2$, $a^{2^{n-1}} = b^2$, $b^{-1}ab = a^{-1}$, $G = N_G^d$;

3) $G = \langle x \rangle \langle b \rangle$, *where* $|x| = p^k$, $p \neq 2$, $|b| = p^m$, $m \geqslant 2$, $k \geqslant m + r$, $Z(G) = \left\langle x^{p^{r+1}} \right\rangle \times \left\langle b^{p^{r+1}} \right\rangle$, $1 \leqslant r \leqslant m-1$, $[x, b] = x^{p^{k-r-1}s} b^{p^{m-1}t}$, $(s, p) = 1$; $N_G^d = \left\langle x^{p^r} \right\rangle \rtimes \langle b \rangle$.

4) $G = \langle x\rangle\langle b\rangle$, $|x| = 2^k$, $|b| = 2^m$, $m > 2$, $k \geqslant m + r$, $1 \leqslant r < m - 1$, $Z(G) = \langle x^{2^{r+1}}\rangle \times \langle b^{2^{r+1}}\rangle$, $[x, b] = x^{2^{k-r-1}s}b^{2^{m-1}t}$, $(s, 2) = 1$, $0 \leqslant t < 2$, $N_G^d = \langle x^{2^r}\rangle \ltimes \langle b\rangle$.

5) $G = \big(\langle x\rangle \ltimes \langle b\rangle\big) \ltimes \langle c\rangle$, $|x| = 2^n$, $n > 3$, $|b| = |c| = 2$, $[x, c] = x^{\pm 2^{n-2}}b$, $[b, c] = [x, b] = x^{2^{n-1}}$, $N_G^d = \big(\langle x^2\rangle \times \langle b\rangle\big) \ltimes \langle c\rangle$;

6) $G = \big(\langle x\rangle \times \langle b\rangle\big) \ltimes \langle c\rangle \ltimes \langle d\rangle$, $|x| = 2^n$, $n > 2$, $|b| = |c| = |d| = 2$, $[x, c] = 1$, $[b, c] = [c, d] = [b, d] = x^{2^{n-1}}$, $d^{-1}xd = x^{-1}$, $N_G^d = \big(\langle x^{2^{n-2}}\rangle \times \langle b\rangle\big) \ltimes \langle c\rangle$;

7) $G = \big(\langle c\rangle \ltimes H\big)\langle y\rangle$, $H = \langle h_1, h_2\rangle$, $|h_1| = |h_2| = 4$, $h_1^2 = h_2^2 = [h_1, h_2]$, $|c| = 4$, $[c, h_1] = c^2$, $[c, h_2] = 1$, $y^2 = h_1$, $[y, h_2] = c^2h_1^2$, $[y, c] = h_2^{\pm 1}$, $N_G^d = \langle c\rangle \ltimes H$;

8) $G = H \cdot Q$, *where* H *is the quaternion group of order 8,* $H = \langle h_1, h_2\rangle$, $|h_1| = |h_2| = 4$, $[h_1, h_2] = h_1^2 = h_2^2$, Q *is a generalized quaternion group,* $Q = \langle y, x\rangle$, $|y| = 2^n$, $n \geqslant 3$, $|x| = 4$, $y^{2^{n-1}} = x^2$, $x^{-1}yx = y^{-1}$, $H \cap Q = E$, $[Q, H] \subseteq \langle x^2, h_1^2\rangle$, $N_G^A = H \times \langle y^{2^{n-2}}\rangle$;

9) $G = \langle x\rangle \ltimes \langle b\rangle$, *where* $|x| = 8, |b| = 2, [x, b] = x^2, N_G^d = \langle x^2\rangle \ltimes \langle b\rangle$;

10) $G = \big(H \times \langle b\rangle\big)\langle a\rangle$, $H = \langle h_1, h_2\rangle$, $|h_1| = 2^n, n > 2$, $h_1^{2^{n-1}} = h_2^2$, $a^2 = h_1^{2^{n-2}}$, $h_2^{-1}h_1h_2 = h_1^{-1}$, $|b| = 2$, $[a, h_1] = a^{4k}$, $[a, h_2] = a^{4l}b$, $[a, b] = a^4; k, l \in \{0, 1\}$, $N_G^d = \langle a\rangle \ltimes \langle b\rangle$;

11) $G = \langle x\rangle\langle b\rangle$, *where* $|x| = 2^k, k \geqslant 4, |b| = 2^m, m \geqslant 2, Z(G) = \langle x^{2^m}\rangle$, $[x, b] = x^{2^{k-m}s}b^{2^{m-1}t}$, $(s, 2) = 1$, $t \in \{0, 1\}$, $N_G^d = \langle x^{2^{m-1}}\rangle \ltimes \langle b\rangle$.

Proof — Let a group G and its norm N_G^d satisfy the condition of the theorem. If $G = N_G^d$, then G is a non-Hamiltonian di-group and is a group of one of the types 1) or 2) of the theorem by Proposition 3. In partiqular, if a group G does not contain decomposable subgroups, then it is a quaternion 2-group of order greater than 8 by Theorem 9 2).

Therefore, let's assume that G contains a decomposable subgroup and $G \neq N_G^d$. By Theorem 11, G is a finite p-group with the non-Dedekind norm N_G^A, and $N_G^d = N_G^A$. Taking into account the description of finite p-groups with the non-Dedekind norm N_G^A (see [3, 12, 18, 19]), we conclude that G is a group of one of the types 3)–11) of Theorem 19. The theorem is proved. □

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