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On the Projective Character Tables of the Maximal Subgroups of M_{11}, M_{12} and Aut(M_{12})

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Abstract

In this paper, the Schur multiplier and irreducible projective character tables $\mathrm{IrrProj}(G, \alpha_i)$ with corresponding factor sets α_i for each maximal subgroup G of the sporadic simple Mathieu groups M_{11}, M_{12} and the automorphism group $\mathrm{Aut}(M_{12})$ of M_{12} are computed. These tables $\mathrm{IrrProj}(G, \alpha_i)$ are obtained from a so-called representation group R of G with the aid of a code which is written in the computational algebra system GAP. In fact, this GAP code can be used to compute the projective character tables for any finite group G on condition that we can find a representation group R of G and its ordinary irreducible characters $\mathrm{Irr}(R)$.

Mathematics Subject Classification (2020): 20C15, 20C25

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1 Introduction

In [1], it is shown that the Schur multipliers of the sporadic simple Mathieu groups M_{11} and M_{12} are trivial and cyclic of order 2, respectively. In addition, the Schur multiplier of the automorphism group $\mathrm{Aut}(M_{12})$ of M_{12} (which is cyclic of order 2) and the projective character tables of M_{12} and $\mathrm{Aut}(M_{12})$ were computed in [5]. In this paper, the Schur multiplier and irreducible projective character tables $\mathrm{IrrProj}(G, \alpha_i)$ with corresponding factor sets α_i for each maximal subgroup G of M_{11}, M_{12} and $\mathrm{Aut}(M_{12})$ are computed. For this purpose, we will compute the sets $\mathrm{IrrProj}(G, \alpha_i)$, $i = 1, 2, \ldots, m$, from a so-called representation group $R \simeq M(G).G$ of the group G, where $M(G)$ denotes the Schur multiplier of the group G and m the number of cohomology classes $[\alpha_i]$ in $M(G)$. A code written in GAP [3] is used for this purpose.

In Section 2, preliminary results on projective character theory are discussed. In Section 3, a proposition and its proof which says that the number $|\mathrm{IrrProj}(G, \alpha)|$ of any set of irreducible projective characters $\mathrm{IrrProj}(G, \alpha)$ with corresponding factor set α of a finite group G is always less or equal to the number $|\mathrm{Irr}(G)|$ of ordinary irreducible characters of G, are given. This proposition and its proof has its origin in a question posed by the current author in [10] and then answered by the author in [16]. From this proposition, a GAP code was written in [17] to determine the number $|\mathrm{IrrProj}(G, \alpha)|$ of irreducible projective characters of a finite group G for any factor set α. Furthermore, this GAP code by [17] is modified and extended in this current paper to compute all the sets $\mathrm{IrrProj}(G, \alpha_i)$ for any given finite group G on condition that one can compute the ordinary character table of the representation group R (Schur cover). As the group G becomes larger it becomes very difficult to compute R and its ordinary irreducible character table in GAP. Readers are also referred to the current author's work in [11], [12], [13] and [14] on the computation of irreducible projective characters of some finite groups.

In Section 4, the GAP code given in Section 3 is used to compute all sets $\mathrm{IrrProj}(G, \alpha_i)$ of irreducible projective characters with associated factor sets α_i for each maximal subgroup of the groups M_{11}, M_{12} and $\mathrm{Aut}(M_{12})$. The details of these sets are tabulated in Section 4.

Computations are carried out with the aid of GAP and notations in both GAP and the ATLAS [2] are followed.

2 Preliminary results on projective characters

In this section, a brief overview of some basic projective character theory pertaining to our study, is given. In what will follows, it will be understood that G is a finite group, $\mathbb{C}$ the field of complex numbers, $\mathbb{C}^*$ the nonzero complex numbers, $GL(n, \mathbb{C})$ the group of nonsingular $n \times n$ matrices over the complex numbers $\mathbb{C}$, $Z(G)$ the center of G, G' the derived subgroup of G, Irr(G) the set of ordinary irreducible characters of G and $\mathrm{IrrProj}(G,\alpha)$ the irreducible projective characters of G with associated factor set α. The author will closely follows the work in [9]. Interested readers are referred to [4], [6], [8], [7] and [15] for a detailed treatment on ordinary and projective character theory.

Definition 1 A *projective representation* of a group G of degree n over the complex numbers is a map $P: G \to GL(n,\mathbb{C})$, such that

(i) $P(1) = I_n$, and

(ii) given $x, y \in G$, there exists $\alpha(x,y) \in \mathbb{C}^*$ such that

$$P(x)P(y) = \alpha(x,y)P(xy).$$

Since multiplication in G and $GL(n,\mathbb{C})$ is associative it follows that

$$\alpha(xy,z)\alpha(x,y) = \alpha(x,yz)\alpha(y,z)$$

for all $x, y, z \in G$. In addition, a map

$$\alpha : G \times G \to \mathbb{C}^*$$

that satisfies this condition is called a *factor set* (or *2-cocycle*) α of G in $\mathbb{C}$. We say that P is a *projective representation with factor set* α. Define $\xi(g) = \mathrm{Trace}(P(g))$ for all $g \in G$, then ξ is called a projective character of G with factor set α. We say that ξ is *irreducible* if P is irreducible. An *irreducible projective representation* P of a group G is essentially defined in a similar way then an ordinary irreducible representation of G.

Definition 2 Two projective representations P_1 and P_2 of G of degree n with factor sets α_1 and α_2 respectively are said to be *projectively equivalent* if there exist a mapping $\phi : G \to \mathbb{C}^*$ and a matrix T in $GL(n,\mathbb{C})$ such that

$$P_1(x) = \phi(g)T^{-1}P_2(g)T, \ \forall x \in G.$$

If P_1 and P_2 are projectively equivalent, then it follows from Definition 2 that, $\forall x, y \in G$,

$$\alpha_2(x,y) = \phi(x)\phi(y)(\phi(xy))^{-1}\alpha_1(x,y)$$

This is an equivalence relation and the equivalence class of the factor set α_1 is denoted by $[\alpha_1]$. The set of all equivalence classes of factor sets of G has a finite abelian group structure, and is called the *Schur multiplier* $M(G)$ (also known as the *second cohomology group* $H^2(G, \mathbb{C}^*)$ of G). The product of two elements $[\alpha_1]$ and $[\alpha_2]$ in $M(G)$ is defined as their pointwise product $[\alpha_1][\alpha_2] = [\alpha_1\alpha_2]$. The class $[1]$ is the identity element of $M(G)$ where 1 is the factor set $1(x, y) = 1$ for all $x, y \in G$ and $[\alpha_1]^{-1} = [\alpha_1^{-1}]$.

Definition 3 A group $C = A.G$ is a *central extension* for G if there exists a homomorphism π from C onto G such that

$$A = \ker(\pi) \leqslant Z(C) \cap C^{'}.$$

In addition, if $A \simeq M(G)$, then we call the central extension C a *representation group* R of G.

Now we will describe how the irreducible projective representations of a group G can be obtained from the ordinary irreducible representations of a central extension $C = A.G$ of G. Let $C = A.G$ be a central extension of the group G with $A = \ker(\pi)$. Let $X = \{x_g | g \in G\}$ be a set of coset representatives of A in C, such that $\pi(x_g) = g$ (one-to-one correspondence of elements of X with the elements of G). Therefore,

$$C = \bigcup_{g \in G} Ax_g.$$

Then, for all $g, h \in G$, let $a(g, h)$ be the unique element in A such that

$$x_g x_h = a(g, h)x_{gh}.$$

Since the product operation to combine two elements in C and G is associative, then it follows that $a(g, h)a(gh, k) = a(g, hk)a(h, k)$ for all $g, h, k \in G$. Now, let λ be a linear character of the abelian group A and put $\alpha(g, h) = \lambda(a(g, h))$ for all $g, h \in G$, then it follows from the relation in the previous sentence that α is a factor set of G. Now, let T be an ordinary irreducible representation of C of degree n and let $P(g) = T(x_g)$ for all $g \in G$, then P is a irreducible projective representation of G with factor set α, i.e. $P(g)P(h) = \lambda(a(g, h))P(gh)$ for all $g, h \in G$. Hence we can formulate the following definition.

Definition 4 A projective representation P of G constructed from an ordinary irreducible representation T of C in the above manner is said to be *linearized* by the ordinary representation T (or *lifted* to C). Furthermore, P is *irreducible* if and only if T is irreducible.

Each irreducible projective representation of G with corresponding factor set α can be linearized by an ordinary irreducible representation of a representation group R of G. So the problem of constructing all irreducible projective characters of a finite group G reduces to that of finding the ordinary irreducible characters of a representation group R of G.

Definition 5 A *covering group* D for G will normally be a quotient $D \simeq R/B$ of a representation group $R = M(G).G$ of G by a subgroup B of $M(G)$. If $M(G)/B$ has order n we sometimes refer to the covering group as a n*-fold cover* of G.

Projective representations of G are found in the representation group R for all the equivalence classes of factors sets in $M(G)$ but however in a n-fold cover D of G only the n equivalence classes which D covers will be represented [4].

Definition 6 An element $x \in G$ is said to be α*-regular* if

$$\alpha(x, g) = \alpha(g, x)$$

for all $g \in C_G(x)$.

Notice that, it is well known that $g \in G$ is α-regular if and only if $\xi(g) \neq 0$ for some $\xi \in \mathrm{IrrProj}(G, \alpha)$ or equivalently that g is α-irregular if and only if $\xi(g) = 0$ for all $\xi \in \mathrm{IrrProj}(G, \alpha)$.

Now, if $x \in G$ is α-regular, then so is every conjugate of x and therefore it is meaningful to speak about α-regular classes of G. The number of $\mathrm{IrrProj}(G, \alpha)$ equals the number of α-regular classes of a group G. Projective characters also satisfy the usual orthogonality relations and have analogues to ordinary characters.

3 A GAP code to compute IrrProj(G, α_i)

In this section, a result in the form of a proposition (due to the author in [16]) is given and it basically tells us under which condition the number $|\mathrm{IrrProj}(G, \alpha)|$ of irreducible projective characters of G with factor set α is equal or strictly less then the number $|\mathrm{Irr}(G)|$ of ordinary irreducible characters of G. The GAP code given in [17] has its origin in this proposition and is computing the number $|\mathrm{IrrProj}(G, \alpha_i)|$ of all irreducible projective characters of G found in each set $\mathrm{IrrProj}(G, \alpha_i)$ of G. In this section, the current author is extending on this GAP code to give a GAP code that can computes directly all the distinct sets $\mathrm{IrrProj}(G, \alpha_i)$ of G, i.e., extracting each distinct set $\mathrm{IrrProj}(G, \alpha_i)$ of G with associated factor set α_i from a representation group R of G.

Proposition 7 (see [16]) *Let $R = M(G).G$ be a representation group of a finite group G, where $M(G)$ denotes the Schur multiplier of G. Then the number of irreducible characters $\mathrm{Irr}(R)$ of R which lies over a linear character θ of $M(G)$ is less or equal to $|Irr(G)|$.*

Proof — The number of irreducible characters $\mathrm{Irr}(R)$ of R which lies over a linear character $\theta \in \mathrm{Irr} M(G)$ is given by

$$\sum_{\chi \in \mathrm{Irr}(R)} \frac{< \chi \downarrow_{M(G)}, \theta >}{\chi(1)}.$$

It is known that the quantity

$$\sum_{\chi \in \mathrm{Irr}(R)} \frac{\chi(x)}{\chi(1)}$$

is non-negative for each $x \in M(G)$, and it is non-zero if x is a commutator in R. For any $\theta \in \mathrm{Irr}(M(G))$, we have

$$\sum_{x \in M(G)} \sum_{\chi \in \mathrm{Irr}(R)} \frac{\chi(x)\theta(x^{-1})}{\chi(1)} \leqslant \sum_{x \in M(G)} \sum_{\chi \in \mathrm{Irr}(R)} \frac{\chi(x)}{\chi(1)}$$
$$= |M(G)||[g]_{R/M(G)}|,$$

where $|[g]_{R/M(G)}|$ is the number of conjugacy classes of $R/M(G) \simeq G$. The last equality follows because the irreducible characters of R with $M(G)$ in their kernels are precisely those which contain the trivial character on the restriction to $M(G)$. Hence

$$\frac{1}{|M(G)|} \sum_{x \in M(G)} \sum_{\chi \in \mathrm{Irr}(R)} \frac{\chi(x)\theta(x^{-1})}{\chi(1)}$$

$$= \sum_{\chi \in \mathrm{Irr}(R)} \frac{1}{|M(G)|} \sum_{x \in M(G)} \frac{\chi(x)\theta(x^{-1})}{\chi(1)}$$

$$= \sum_{\chi \in \mathrm{Irr}(R)} \frac{< \chi \downarrow_{M(G)}, \theta >}{\chi(1)} \leqslant |[g]_{R/M(G)}| = |\mathrm{Irr}(G)|.$$

Furthermore, if there is a non-identity element $x \in M(G) \setminus \ker(\theta)$ which is a commutator in R, then the inequality becomes strict. □

Now the below GAP code can computes directly all the distinct sets $\mathrm{IrrProj}(G, \alpha_i)$ of G from a suitable representation group R (full covering group) of G. Especially, if the finite group G has a relatively small order then the representation group (Schur cover) of G can be computed easily in GAP. But if the group G becomes too large then GAP experiences difficulties to compute the Schur cover of G and then we have to employ additional techniques in computing these sets $\mathrm{IrrProj}(G, \alpha_i)$. Fortunately, the orders of the groups under consideration in this paper are not too large and the aforementioned GAP code is implemented successfully to compute all the sets $\mathrm{IrrProj}(G, \alpha_i)$ of the groups.

The first and most important part of the GAP code will results in $|\mathrm{Irr}(M(G)|$ blocks coming from a representation group R (denoted as "Source(f)" in the below code), where each block will contains one of the sets $\mathrm{IrrProj}(G, \alpha_i)$, $i = 1, 2, \ldots, |\mathrm{Irr}(M(G)|$. In this code the Schur multiplier $M(G)$ of G is labelled as "z". The crucial part of the code is the line of the code starting with "n" which is based on the fact that each $\phi \in \mathrm{IrrProj}(G, \alpha_i)$ can be linearized (as explained in Section 2) to an ordinary irreducible character χ of $R = M(G).G$ such that $\lambda \in \mathrm{Irr}(M(G))$ is an irreducible constituent of $\chi_{M(G)}$, that is,

$$< \chi_{M(G)}, \lambda > \neq 0.$$

We also say that λ lies under χ or equivalent by the Frobenius reciprocity that χ lies over λ. In this manner we can obtain all the sets $\mathrm{IrrProj}(G, \alpha_i)$ of G associated with a factor set α_i.

```
gap> h := G;;
# G is a permutation group with generators found
# in [18] or can be generated in GAP
gap> f := EpimorphismSchurCover(h);;
gap> z := Kernel(f);;
gap> x := Source(f);;
gap> I1:=ImagesSource(f);;
```

```
# Quotient group I1 ≃ G
gap> t:=Irr(I1);;
gap> 2t:=Irr(x);;
gap> F:=FusionConjugacyClassesOp(f);
gap> map:=ProjectionMap(F);
gap> N:=[];
gap> for i in [1..Size(Irr(z))] do
> n:=Filtered(Irr(x),chi->
> not IsZero(ScalarProduct(RestrictedClassFunction
> (chi,z),Irr(z)[i])));
> s:=List(n,x->x{map});
> Add(N,s);
> od;
gap> N;
# The sets IrrProj(G,α_i^-1) partitioned into Size(Irr(z))
# blocks according to factor set α_i.
```

The following part of the GAP code will display the irreducible projective character tables with associated factor sets individually, for each maximal subgroup of M_{11}, M_{12} and $\mathrm{Aut}(M_{12})$.

```
gap> Cen:=SizesCentralizers(CharacterTable(I1));
gap> Cl:=OrdersClassRepresentatives(CharacterTable(I1));
gap> for i in [1..Size(Irr(z))] do
> ct:=function()local ct ;ct:=rec();
> ct.SizesCentralizers:=Cen;;
> ct.OrdersClassRepresentatives:=Cl;;
> ct.Irr:=N[i];;
> ct.UnderlyingCharacteristic:=0;ct.Id:="G";
> ConvertToLibraryCharacterTableNC(ct);return ct;end;
> ct:=ct();
> SetInfoLevel(InfoCharacterTable,2);
> Display(ct);
> od;
```

4 The sets IrrProj(G,α_i) for the maximal subgroups of M_{11}, M_{12} and Aut(M_{12})

In this section, all the sets $\mathrm{IrrProj}(G,\alpha_i)$ of irreducible projective characters for each maximal subgroup G of M_{11}, M_{12} and $\mathrm{Aut}(M_{12})$ are computed with the GAP code presented in Section 3 of this paper and they are found in the tables below.

For example, the maximal subgroup $G = 2^{1+4}{:}S_3$ of M_{12} has a Schur multiplier $M(G) \simeq 2^2$ which is isomorphic to the elementary abelian group 2^2 of order 4. Therefore, $M(G)$ contains three cohomology classes $[\alpha_i]$ of order 2 and the trivial class [1]. Hence we have three sets of projective characters $\mathrm{IrrProj}(G, \alpha_i)$ lying above the non-trivial factor sets $\alpha_i \in [\alpha_i]$, $i = 2, 3, 4$, such that $\alpha_i^2 \sim 1$ and where the set of ordinary irreducible characters $\mathrm{Irr}(G)$ is associated with the identity class [1] of $M(G)$. Using the GAP code in Section 3, the representation group $R \simeq 2^2.(2^{1+4}{:}S_3)$ of G will be subdivided into 4 blocks and where each block contains a set $\mathrm{IrrProj}(G, \alpha_i)$. The sizes of the sets $\mathrm{IrrProj}(G, \alpha_i)$, $i = 1, 2, 3, 4$, are 13, 10, 8 and 6. The set $\mathrm{IrrProj}(G, \alpha_1)$ with 13 projective characters is the ordinary irreducible characters of G whereas the other sets are the irreducible projective characters of G associated with the nontrivial factor sets α_i. These characters are found in Table 2 with the first block the set $\mathrm{Irr}(G)$ and the other three blocks at the bottom are the sets $\mathrm{IrrProj}(G, \alpha_i)$ with $\alpha_i^2 \sim 1$, $i = 2, 3, 4$ (as in [4]). The first two rows of Table 2 list the class orders and centralizer sizes for the conjugacy classes of G. For the interested reader, the sets $\mathrm{IrrProj}(G, \alpha_i)$ of Table 2 can be view in GAP by using the second part of the GAP code in Section 3. All the other sets $\mathrm{IrrProj}(G, \alpha_i)$ for each maximal subgroup G (with nontrivial Schur multipliers) of M_{11}, M_{12} and $\mathrm{Aut}(M_{12})$ are tabulated below except the ones for the groups $L_2(11), L_2(11){:}2, S_5, M_{11}$ and M_{12} which can be found in the ATLAS. These tables have the same format as Table 2. The ordinary character tables of the maximal subgroups with trivial Schur multipliers are uploaded in the GAP library. In addition, the information concerning the structures of the Schur Multipliers $M(G)$ and the number of sets $\mathrm{IrrProj}(G, \alpha_i)$ for all the maximal subgroups of M_{11}, M_{12} and $\mathrm{Aut}(M_{12})$ are summarized in Table 1. Note that the structure of the Schur Multiplier $M(G)$ for each group G can also be determined by the GAP command `AbelianInvariantsMultiplier(G).`

Also, it is worthwhile to mention that the Schur multiplier of the maximal subgroup $A_6.2_3$ of M_{11} is cyclic of order three and will have two sets $\mathrm{IrrProj}(A_6.2_3, \alpha_i)$ with nontrivial factor sets α_2 and α_2^2 of order three. The trivial factor set $\alpha_1 = \alpha_2^3 = 1$ is associated with the ordinary irreducible characters $\mathrm{Irr}(A_6.2_3)$ of $A_6.2_3$. Since $\alpha_2^2 = \alpha_2^{-1} = \overline{\alpha_2}$ the entries of the set $\mathrm{IrrProj}(A_6.2_3, \alpha_2)$ of $A_6.2_3$

in the second block of Table 3 are just the complex conjugates of the entries of the set IrrProj$(A_6.2_3, \alpha_2^2)$ given in the third block of Table 3. Therefore, we will found that in the ATLAS only one set IrrProj$(A_6.2_3, \alpha_i)$ of irreducible projective characters with nontrivial factor set α_i is given for the automorphism group $A_6.2_3$ of A_6. The sets IrrProj$(A_6.2_3, \alpha_i)$ in Table 3 which were obtained with the GAP code in Section 3 confirmed those of $A_6.2_3$ found in the ATLAS.

Table 1: Schur Multipliers $M(G)$ and $|\text{IrrProj}(G, \alpha_i)|$ of maximal subgroups G of M_{11}, M_{12} and $\text{Aut}(M_{12})$

Maximal subgroups of M_{11}	$\|G\|$	$[M_{11}:G]$	$M(G)$	$\|\text{IrrProj}(G, \alpha_i)\|$
$M_{10} \simeq A_6.2_3$	720	11	3	[8,7,7]
$L_2(11)$	660	12	2	[8,7]
$M_9{:}2 \simeq 3^2{:}Q_8.2$	144	55	1	[9]
S_5	120	66	2	[7,5]
$M_8{:}S_3 \simeq 2^{\cdot}S_4$	48	165	1	[8]
Maximal subgroups of M_{12}	$\|G\|$	$[M_{12}:G]$	$M(G)$	$\|\text{IrrProj}(G, \alpha_i)\|$
M_{11}	7920	12	1	[10]
M_{11}	7920	12	1	[10]
$A_6{}^{\cdot}2^2 \simeq M_{10}{:}2$	1440	66	2	[13,10]
$A_6{}^{\cdot}2^2 \simeq M_{10}{:}2$	1440	66	2	[13,10]
$L_2(11)$	660	144	2	[8,7]
$3^2{:}2S_4 \simeq M_9{:}3$	432	220	1	[11]
$3^2{:}2S_4 \simeq M_9{:}3$	432	220	1	[11]
$2 \times S_5$	240	396	2^2	[14,10,5,4]
$2^{1+4}{:}S_3 \simeq M_8.S_4$	192	495	2^2	[13,6,10,8]
$4^2{:}D_{12}$	192	495	2^2	[14,7,7,8]
$A_4 \times S_3$	72	1320	2	[12,9]
Maximal subgroups of $\text{Aut}(M_{12})$	$\|G\|$	$[\text{Aut}(M_{12}):G]$	$M(G)$	$\|\text{IrrProj}(G, \alpha_i)\|$
M_{12}	95040	2	2	[15,11]
$L_2(11){:}2$	1320	144	2	[13,11]
$L_2(11){:}2$	1320	144	2	[13,11]
$(2^2 \times A_5){:}2$	480	396	2^2	[19,5,14,7]
$(2^{1+4}{:}S_3).2$	384	495	2^2	[17,13,14,10]
$(4^2{:}D_{12}).2$	384	495	2^2	[16,7,11,8]
$3^{1+2}{:}D_8$	216	880	2	[13,10]
$S_4 \times S_3$	144	1320	2^2	[15,6,9,9]
S_5	120	1584	2	[7,5]

Table 2: $\mathrm{IrrProj}(G, \alpha_i)$ of $2^{1+4}{:}S_3$

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$

$[g]_G$	1a	4a	3a	2a	2b	2c	8a	8b	6a	4b	4c	4d	2d
$\lvert C_G(g)\rvert$	192	16	6	16	32	8	8	8	6	32	32	16	192
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	1	-1	-1	-1	1	1	1	-1	1
χ_3	2	0	-1	2	2	0	0	0	-1	2	2	0	2
χ_4	3	1	0	-1	-1	-1	-1	1	0	3	-1	1	3
χ_5	3	-1	0	-1	-1	1	1	-1	0	3	-1	-1	3
χ_6	3	-1	0	-1	3	-1	1	1	0	-1	-1	-1	3
χ_7	3	1	0	-1	3	1	-1	-1	0	-1	-1	1	3
χ_8	3	1	0	-1	-1	-1	1	-1	0	-1	3	1	3
χ_9	3	-1	0	-1	-1	1	-1	1	0	-1	3	-1	3
χ_{10}	4	-2	1	0	0	0	0	0	-1	0	0	2	-4
χ_{11}	4	2	1	0	0	0	0	0	-1	0	0	-2	-4
χ_{12}	6	0	0	2	-2	0	0	0	0	-2	-2	0	6
χ_{13}	8	0	-1	0	0	0	0	0	1	0	0	0	-8
χ_1	4	0	1	0	0	0	0	-2	1	0	0	0	-4
χ_2	4	0	1	0	0	0	0	2	1	0	0	0	-4
χ_3	4	0	1	0	0	0	-2	0	-1	0	0	0	4
χ_4	4	0	1	0	0	0	2	0	-1	0	0	0	4
χ_5	8	0	-1	0	0	0	0	0	1	0	0	0	8
χ_6	8	0	-1	0	0	0	0	0	-1	0	0	0	-8
χ_1	2	A	-1	0	0	0	0	A	1	-2	0	-A	-2
χ_2	2	-A	-1	0	0	0	0	-A	1	-2	0	A	-2
χ_3	2	A	-1	0	0	0	A	0	-1	0	2	A	2
χ_4	2	-A	-1	0	0	0	-A	0	-1	0	2	-A	2
χ_5	4	0	1	0	0	0	0	0	-1	-4	0	0	-4
χ_6	4	0	1	0	0	0	0	0	1	0	4	0	4
χ_7	6	A	0	0	0	0	-A	0	0	0	-2	A	6
χ_8	6	-A	0	0	0	0	A	0	0	0	-2	-A	6
χ_9	6	A	0	0	0	0	0	-A	0	2	0	-A	-6
χ_{10}	6	-A	0	0	0	0	0	A	0	2	0	A	-6
χ_1	2	0	-1	0	2	0	A	A	1	0	0	0	2
χ_2	2	0	-1	0	2	0	-A	-A	1	0	0	0	2
χ_3	4	0	1	0	4	0	0	0	-1	0	0	0	4
χ_4	4	0	1	0	0	-2	0	0	1	0	0	0	-4
χ_5	4	0	1	0	0	2	0	0	1	0	0	0	-4
χ_6	6	0	0	0	-2	0	-A	A	0	0	0	0	6
χ_7	6	0	0	0	-2	0	A	-A	0	0	0	0	6
χ_8	8	0	-1	0	0	0	0	0	-1	0	0	0	-8

where $A = -\sqrt{2}i$

Table 3: IrrProj(G,α_i) of $A_6.2_3$
$\alpha_1 = \alpha_2^3 = \alpha_3^3 = 1$

$[g]_G$	1a	4a	5a	2a	8a	8b	3a	4b
$\lvert C_G(g)\rvert$	720	4	5	16	8	8	9	8
χ_1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	-1	-1	1	1
χ_3	9	1	-1	1	-1	-1	0	1
χ_4	9	-1	-1	1	1	1	0	1
χ_5	10	0	0	2	0	0	1	-2
χ_6	10	0	0	-2	A	-A	1	0
χ_7	10	0	0	-2	-A	A	1	0
χ_8	16	0	1	0	0	0	-2	0
χ_1	6	0	1	-2	0	0	0	2
χ_2	6	0	1	2	A	-A	0	0
χ_3	6	0	1	2	-A	A	0	0
χ_4	9	1	-1	1	-1	-1	0	1
χ_5	9	-1	-1	1	1	1	0	1
χ_6	15	-1	0	-1	-1	-1	0	-1
χ_7	15	1	0	-1	1	1	0	-1
χ_1	6	0	1	-2	0	0	0	2
χ_2	6	0	1	2	A	-A	0	0
χ_3	6	0	1	2	-A	A	0	0
χ_4	9	1	-1	1	-1	-1	0	1
χ_5	9	-1	-1	1	1	1	0	1
χ_6	15	-1	0	-1	-1	-1	0	-1
χ_7	15	1	0	-1	1	1	0	-1

where A = $-\sqrt{2}i$

Table 4: IrrProj(G, α_i) of $A_6{}^{\cdot}2^2$

$$\alpha_1 = \alpha_2^2 = 1$$

$[g]_G$	1a	2a	4a	8a	8b	2b	5a	10a	4b	4c	3a	2c	6a
$\lvert C_G(g)\rvert$	1440	32	16	8	8	40	10	10	16	8	18	48	6
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	-1	1	-1	1	-1	1	1	1
χ_3	1	1	1	-1	1	-1	1	-1	-1	1	1	-1	-1
χ_4	1	1	1	1	-1	1	1	1	-1	-1	1	-1	-1
χ_5	9	1	1	1	-1	-1	-1	-1	-1	1	0	3	0
χ_6	9	1	1	1	1	-1	-1	-1	1	-1	0	-3	0
χ_7	9	1	1	-1	-1	1	-1	1	1	1	0	-3	0
χ_8	9	1	1	-1	1	1	-1	1	-1	-1	0	3	0
χ_9	10	2	-2	0	0	0	0	0	2	0	1	2	-1
χ_{10}	10	2	-2	0	0	0	0	0	-2	0	1	-2	1
χ_{11}	16	0	0	0	0	-4	1	1	0	0	-2	0	0
χ_{12}	16	0	0	0	0	4	1	-1	0	0	-2	0	0
χ_{13}	20	-4	0	0	0	0	0	0	0	0	2	0	0
χ_1	2	2	-2	0	0	0	2	0	0	0	2	0	0
χ_2	10	2	2	0	0	0	0	0	0	0	1	4	1
χ_3	10	2	2	0	0	0	0	0	0	0	1	-4	-1
χ_4	10	-2	0	A	A	0	0	0	0	0	1	2	-1
χ_5	10	-2	0	-A	-A	0	0	0	0	0	1	2	-1
χ_6	10	-2	0	-A	A	0	0	0	0	0	1	-2	1
χ_7	10	-2	0	A	-A	0	0	0	0	0	1	-2	1
χ_8	16	0	0	0	0	0	1	B	0	0	-2	0	0
χ_9	16	0	0	0	0	0	1	-B	0	0	-2	0	0
χ_{10}	18	2	-2	0	0	0	-2	0	0	0	0	0	0

where A = $-\sqrt{2}$, B = $-\sqrt{5}$

Table 5: $\mathrm{IrrProj}(G, \alpha_i)$ of $2 \times S_5$

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$

$[g]_G$	1a	2a	3a	2b	4a	6a	5a	2c	2d	6b	2e	4b	6c	10a
$\lvert C_G(g)\rvert$	240	24	12	16	8	12	10	240	24	12	16	8	12	10
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	-1	-1	1	-1	1	-1	-1	1	1	-1
χ_3	1	-1	1	1	-1	-1	1	1	-1	1	1	-1	-1	1
χ_4	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
χ_5	4	-2	1	0	0	1	-1	-4	2	-1	0	0	-1	1
χ_6	4	-2	1	0	0	1	-1	4	-2	1	0	0	1	-1
χ_7	4	2	1	0	0	-1	-1	-4	-2	-1	0	0	1	1
χ_8	4	2	1	0	0	-1	-1	4	2	1	0	0	-1	-1
χ_9	5	-1	-1	1	1	-1	0	-5	1	1	-1	-1	1	0
χ_{10}	5	-1	-1	1	1	-1	0	5	-1	-1	1	1	-1	0
χ_{11}	5	1	-1	1	-1	1	0	-5	-1	1	-1	1	-1	0
χ_{12}	5	1	-1	1	-1	1	0	5	1	-1	1	-1	1	0
χ_{13}	6	0	0	-2	0	0	1	6	0	0	-2	0	0	1
χ_{14}	6	0	0	-2	0	0	1	-6	0	0	2	0	0	-1
χ_1	4	0	2	0	0	0	-1	4	0	2	0	0	0	-1
χ_2	4	0	2	0	0	0	-1	-4	0	-2	0	0	0	1
χ_3	4	0	-1	0	0	B	-1	-4	0	1	0	0	-B	1
χ_4	4	0	-1	0	0	-B	-1	-4	0	1	0	0	B	1
χ_5	4	0	-1	0	0	B	-1	4	0	-1	0	0	B	-1
χ_6	4	0	-1	0	0	-B	-1	4	0	-1	0	0	-B	-1
χ_7	6	0	0	0	A	0	1	-6	0	0	0	-A	0	-1
χ_8	6	0	0	0	-A	0	1	-6	0	0	0	A	0	-1
χ_9	6	0	0	0	A	0	1	6	0	0	0	A	0	1
χ_{10}	6	0	0	0	-A	0	1	6	0	0	0	-A	0	1
χ_1	2	0	2	2	0	0	2	0	0	0	0	0	0	0
χ_2	6	0	0	-2	0	0	1	0	0	0	0	0	0	C
χ_3	6	0	0	-2	0	0	1	0	0	0	0	0	0	-C
χ_4	8	0	2	0	0	0	-2	0	0	0	0	0	0	0
χ_5	10	0	-2	2	0	0	0	0	0	0	0	0	0	0
χ_1	4	0	2	0	0	0	-1	0	0	0	0	0	0	C
χ_2	4	0	2	0	0	0	-1	0	0	0	0	0	0	-C
χ_3	8	0	-2	0	0	0	-2	0	0	0	0	0	0	0
χ_4	12	0	0	0	0	0	2	0	0	0	0	0	0	0

where A = $-\sqrt{2}i$, B = $-\sqrt{3}i$, C= $-\sqrt{5}$.

Table 6: IrrProj(G, α_i) of 4^2:D_{12}

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$

$[g]_G$	1a	2a	2b	3a	4a	2c	2d	6a	2e	8a	4b	4c	8b	4d
$\lvert C_G(g)\rvert$	192	48	16	6	32	64	16	6	16	8	16	32	8	16
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1
χ_3	1	-1	1	1	1	1	-1	-1	-1	1	1	1	-1	-1
χ_4	1	1	-1	1	1	1	-1	1	1	-1	-1	1	-1	-1
χ_5	2	-2	0	-1	2	2	0	1	-2	0	0	2	0	0
χ_6	2	2	0	-1	2	2	0	-1	2	0	0	2	0	0
χ_7	3	-3	-1	0	-1	3	1	0	1	1	-1	-1	-1	1
χ_8	3	-3	1	0	-1	3	-1	0	1	-1	1	-1	1	-1
χ_9	3	3	-1	0	-1	3	-1	0	-1	1	-1	-1	1	-1
χ_{10}	3	3	1	0	-1	3	1	0	-1	-1	1	-1	-1	1
χ_{11}	6	0	-2	0	2	-2	0	0	0	0	2	-2	0	0
χ_{12}	6	0	0	0	-2	-2	-2	0	0	0	0	2	0	2
χ_{13}	6	0	0	0	-2	-2	2	0	0	0	0	2	0	-2
χ_{14}	6	0	2	0	2	-2	0	0	0	0	-2	-2	0	0
χ_1	2	-2	0	-1	0	-2	0	1	0	A	0	0	-A	0
χ_2	2	-2	0	-1	0	-2	0	1	0	-A	0	0	A	0
χ_3	2	2	0	-1	0	-2	0	-1	0	A	0	0	A	0
χ_4	2	2	0	-1	0	-2	0	-1	0	-A	0	0	-A	0
χ_5	4	-4	0	1	0	-4	0	-1	0	0	0	0	0	0
χ_6	4	4	0	1	0	-4	0	1	0	0	0	0	0	0
χ_7	12	0	0	0	0	4	0	0	0	0	0	0	0	0
χ_1	4	0	0	-2	0	-4	0	0	0	0	0	0	0	0
χ_2	4	0	0	1	0	-4	0	B	0	0	0	0	0	0
χ_3	4	0	0	1	0	-4	0	-B	0	0	0	0	0	0
χ_4	6	0	0	0	0	2	0	0	C	-A	0	0	D	0
χ_5	6	0	0	0	0	2	0	0	C	A	0	0	-D	0
χ_6	6	0	0	0	0	2	0	0	-C	-A	0	0	-D	0
χ_7	6	0	0	0	0	2	0	0	-C	A	0	0	D	0
χ_1	2	0	0	2	2	2	0	0	0	0	0	2	0	0
χ_2	2	0	0	-1	2	2	0	B	0	0	0	2	0	0
χ_3	2	0	0	-1	2	2	0	-B	0	0	0	2	0	0
χ_4	6	0	0	0	-2	6	0	0	0	0	0	-2	0	0
χ_5	6	0	0	0	2	-2	0	0	0	-2	0	-2	0	0
χ_6	6	0	0	0	2	-2	0	0	0	2	0	-2	0	0
χ_7	6	0	0	0	-2	-2	0	0	0	0	0	2	C	0
χ_8	6	0	0	0	-2	-2	0	0	0	0	0	2	-C	0

where A = $\sqrt{2}i$, B = $-\sqrt{3}i$, C= $-2i$, D = $\sqrt{2}$

Table 7: IrrProj(G,α_i) of $(2^2\times A_5)$:2

$$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$$

$[g]_G$	1a	4a	4b	3a	2a	10a	2b	4c	2c	6a	6b	10b	6c	5a	10c	2d	12a	2e	2f
$\lvert C_G(g)\rvert$	480	8	8	24	32	20	16	24	32	12	24	20	12	20	20	24	12	480	240
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	1	1	-1	-1	1	1
χ_3	1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	1	1	-1
χ_4	1	-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	1	1	1	-1	1	-1
χ_5	2	0	0	2	-2	0	0	0	2	0	-2	0	0	2	-2	0	0	-2	0
χ_6	4	0	0	1	0	-1	0	-2	0	1	1	-1	1	-1	-1	-2	1	4	4
χ_7	4	0	0	1	0	1	0	-2	0	-1	1	1	-1	-1	-1	2	1	4	-4
χ_8	4	0	0	1	0	1	0	2	0	1	1	1	-1	-1	-1	-2	-1	4	-4
χ_9	4	0	0	1	0	-1	0	2	0	-1	1	-1	1	-1	-1	2	-1	4	4
χ_{10}	5	1	-1	-1	1	0	-1	-1	1	1	-1	0	1	0	0	1	-1	5	-5
χ_{11}	5	-1	-1	-1	1	0	1	1	1	1	-1	0	-1	0	0	1	1	5	5
χ_{12}	5	1	1	-1	1	0	1	-1	1	-1	-1	0	-1	0	0	-1	-1	5	5
χ_{13}	5	-1	1	-1	1	0	-1	1	1	-1	-1	0	1	0	0	-1	1	5	-5
χ_{14}	6	0	0	0	-2	-1	2	0	-2	0	0	-1	0	1	1	0	0	6	-6
χ_{15}	6	0	0	0	-2	1	-2	0	-2	0	0	1	0	1	1	0	0	6	6
χ_{16}	6	0	0	0	2	D	0	0	-2	0	0	-D	0	1	-1	0	0	-6	0
χ_{17}	6	0	0	0	2	-D	0	0	-2	0	0	D	0	1	-1	0	0	-6	0
χ_{18}	8	0	0	2	0	0	0	0	0	0	-2	0	0	-2	2	0	0	-8	0
χ_{19}	10	0	0	-2	-2	0	0	0	2	0	2	0	0	0	0	0	0	-10	0
χ_1	8	0	0	-4	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0
χ_2	8	0	0	2	0	0	0	0	0	0	0	0	0	-2	0	0	G	0	0
χ_3	8	0	0	2	0	0	0	0	0	0	0	0	0	-2	0	0	-G	0	0
χ_4	12	A	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
χ_5	12	-A	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
χ_1	4	0	0	-2	0	-1	0	0	0	0	-2	-1	-2	-1	-1	0	0	4	4
χ_2	4	0	0	-2	0	1	0	0	0	0	-2	1	2	-1	-1	0	0	4	-4
χ_3	4	0	0	-2	0	D	0	0	0	0	2	-D	0	-1	1	0	0	-4	0
χ_4	4	0	0	-2	0	-D	0	0	0	0	2	D	0	-1	1	0	0	-4	0
χ_5	4	0	0	1	0	-1	0	0	0	F	1	-1	1	-1	-1	0	F	4	4
χ_6	4	0	0	1	0	-1	0	0	0	-F	1	-1	1	-1	-1	0	-F	4	4
χ_7	4	0	0	1	0	1	0	0	0	-F	1	1	-1	-1	-1	0	F	4	-4
χ_8	4	0	0	1	0	1	0	0	0	F	1	1	-1	-1	-1	0	-F	4	-4
χ_9	6	B	-B	0	0	-1	0	0	0	0	0	-1	0	1	1	0	0	6	-6
χ_{10}	6	-B	B	0	0	-1	0	0	0	0	0	-1	0	1	1	0	0	6	-6
χ_{11}	6	-B	-B	0	0	1	0	0	0	0	0	1	0	1	1	0	0	6	6
χ_{12}	6	B	B	0	0	1	0	0	0	0	0	1	0	1	1	0	0	6	6
χ_{13}	8	0	0	2	0	0	0	0	0	0	-2	0	0	-2	2	0	0	-8	0
χ_{14}	12	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	-12	0
χ_1	2	C	0	2	0	0	0	C	2	0	0	0	0	2	0	0	-C	0	0
χ_2	2	-C	0	2	0	0	0	-C	2	0	0	0	0	2	0	0	C	0	0
χ_3	8	0	0	2	0	0	0	E	0	0	0	0	0	-2	0	0	C	0	0
χ_4	8	0	0	2	0	0	0	-E	0	0	0	0	0	-2	0	0	-C	0	0
χ_5	10	-C	0	-2	0	0	0	C	2	0	0	0	0	0	0	0	-C	0	0
χ_6	10	C	0	-2	0	0	0	-C	2	0	0	0	0	0	0	0	C	0	0
χ_7	12	0	0	0	0	0	0	0	-4	0	0	0	0	2	0	0	0	0	0

where A = -2i, B = $\sqrt{2}$i, C = -$\sqrt{2}$, D = $\sqrt{5}$,
E = -2$\sqrt{2}$, F = -$\sqrt{3}$i, G = -$\sqrt{6}$i

Table 8: IrrProj(G,α_i) of $(2^{1+4}{:}S_3).2$

$$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$$

$[g]_G$	1a	4a	2a	3a	4b	2b	12a	2c	4c	6a	2d	4d	12b	4e	8a	2e	4f
$\lvert C_G(g)\rvert$	384	16	16	12	32	64	12	16	8	12	384	16	12	16	8	32	48
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	1	1	-1	-1	1	1	1	1	-1	-1	-1	1	-1
χ_3	1	1	-1	1	1	1	-1	1	-1	1	1	-1	-1	-1	1	1	-1
χ_4	1	-1	-1	1	1	1	1	-1	-1	1	1	-1	1	1	-1	1	1
χ_5	2	0	0	-1	2	2	1	0	0	-1	2	0	1	-2	0	2	-2
χ_6	2	0	0	-1	2	2	-1	0	0	-1	2	0	-1	2	0	2	2
χ_7	3	-1	-1	0	-1	3	0	-1	1	0	3	-1	0	-1	1	-1	3
χ_8	3	1	1	0	-1	3	0	1	-1	0	3	1	0	-1	-1	-1	3
χ_9	3	-1	1	0	-1	3	0	-1	-1	0	3	1	0	1	1	-1	-3
χ_{10}	3	1	-1	0	-1	3	0	1	1	0	3	-1	0	1	-1	-1	-3
χ_{11}	6	-2	0	0	2	-2	0	2	0	0	6	0	0	0	0	-2	0
χ_{12}	6	0	2	0	-2	-2	0	0	0	0	6	-2	0	0	0	2	0
χ_{13}	6	0	-2	0	-2	-2	0	0	0	0	6	2	0	0	0	2	0
χ_{14}	6	2	0	0	2	-2	0	-2	0	0	6	0	0	0	0	-2	0
χ_{15}	8	0	0	2	0	0	0	0	0	-2	-8	0	0	0	0	0	0
χ_{16}	8	0	0	-1	0	0	A	0	0	1	-8	0	-A	0	0	0	0
χ_{17}	8	0	0	-1	0	0	-A	0	0	1	-8	0	A	0	0	0	0
χ_1	4	0	0	-2	0	4	0	0	0	-2	4	0	0	0	0	0	0
χ_2	4	0	0	1	0	0	1	2	0	-1	-4	-2	1	0	0	0	-2
χ_3	4	0	0	1	0	0	-1	-2	0	-1	-4	-2	-1	0	0	0	2
χ_4	4	0	0	1	0	0	1	-2	0	-1	-4	2	1	0	0	0	-2
χ_5	4	0	0	1	0	0	-1	2	0	-1	-4	2	-1	0	0	0	2
χ_6	4	0	0	1	0	4	A	0	0	1	4	0	-A	0	0	0	0
χ_7	4	0	0	1	0	4	-A	0	0	1	4	0	A	0	0	0	0
χ_8	6	0	0	0	0	-2	0	0	B	0	6	0	0	-2	B	0	0
χ_9	6	0	0	0	0	-2	0	0	-B	0	6	0	0	-2	-B	0	0
χ_{10}	6	0	0	0	0	-2	0	0	-B	0	6	0	0	2	B	0	0
χ_{11}	6	0	0	0	0	-2	0	0	B	0	6	0	0	2	-B	0	0
χ_{12}	8	0	0	-1	0	0	-1	0	0	1	-8	0	-1	0	0	0	-4
χ_{13}	8	0	0	-1	0	0	1	0	0	1	-8	0	1	0	0	0	4
χ_1	2	0	0	2	2	-2	0	0	0	-2	-2	0	0	0	0	-2	0
χ_2	2	0	0	-1	2	-2	A	0	0	1	-2	0	-A	0	0	-2	0
χ_3	2	0	0	-1	2	-2	-A	0	0	1	-2	0	A	0	0	-2	0
χ_4	4	-2	2	1	0	0	1	0	0	1	4	0	1	0	0	0	-2
χ_5	4	-2	-2	1	0	0	-1	0	0	1	4	0	-1	0	0	0	2
χ_6	4	2	-2	1	0	0	1	0	0	1	4	0	1	0	0	0	-2
χ_7	4	2	2	1	0	0	-1	0	0	1	4	0	-1	0	0	0	2
χ_8	6	0	0	0	-2	-6	0	0	0	0	-6	0	0	0	0	2	0
χ_9	6	0	0	0	-2	2	0	0	-2	0	-6	0	0	0	0	-2	0
χ_{10}	6	0	0	0	-2	2	0	0	2	0	-6	0	0	0	0	-2	0
χ_{11}	6	0	0	0	2	2	0	0	0	0	-6	0	0	0	-2	2	0
χ_{12}	6	0	0	0	2	2	0	0	0	0	-6	0	0	0	2	2	0
χ_{13}	8	0	0	-1	0	0	-1	0	0	-1	8	0	-1	0	0	0	-4
χ_{14}	8	0	0	-1	0	0	1	0	0	-1	8	0	1	0	0	0	4
χ_1	2	0	0	-1	0	-2	1	0	B	1	-2	0	1	0	B	0	-2
χ_2	2	0	0	-1	0	-2	1	0	-B	1	-2	0	1	0	-B	0	-2
χ_3	2	0	0	-1	0	-2	-1	0	-B	1	-2	0	-1	0	B	0	2
χ_4	2	0	0	-1	0	-2	-1	0	B	1	-2	0	-1	0	-B	0	2
χ_5	4	0	0	1	0	-4	-1	0	0	-1	-4	0	-1	0	0	0	-4
χ_6	4	0	0	1	0	-4	1	0	0	-1	-4	0	1	0	0	0	4
χ_7	8	0	0	2	0	0	0	0	0	2	8	0	0	0	0	0	0
χ_8	8	0	0	-1	0	0	A	0	0	-1	8	0	-A	0	0	0	0
χ_9	8	0	0	-1	0	0	-A	0	0	-1	8	0	A	0	0	0	0
χ_{10}	12	0	0	0	0	4	0	0	0	0	-12	0	0	0	0	0	0

where A = $\sqrt{3}$, B = -$\sqrt{2}$i

Table 9: IrrProj(G,α_i) for $(4^2{:}D_{12}).2$

$$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$$

$[g]_G$	1a	2a	2b	2c	3a	4a	2d	4b	8a	4c	6a	4d	6b	2e	4e	6c
$\|C_G(g)\|$	384	16	48	96	12	32	128	8	8	16	12	16	12	32	8	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	-1
χ_3	1	1	-1	1	1	1	1	-1	1	1	-1	-1	1	1	-1	-1
χ_4	1	-1	1	1	1	1	1	-1	-1	-1	1	1	1	1	-1	1
χ_5	2	0	-2	2	-1	2	2	0	0	0	1	-2	-1	2	0	1
χ_6	2	0	2	2	-1	2	2	0	0	0	-1	2	-1	2	0	-1
χ_7	2	0	0	-2	2	2	2	0	0	0	0	0	-2	-2	0	0
χ_8	2	0	0	-2	-1	2	2	0	0	0	D	0	1	-2	0	-D
χ_9	2	0	0	-2	-1	2	2	0	0	0	-D	0	1	-2	0	D
χ_{10}	3	-1	-3	3	0	-1	3	1	1	-1	0	1	0	-1	-1	0
χ_{11}	3	1	-3	3	0	-1	3	-1	-1	1	0	1	0	-1	1	0
χ_{12}	3	-1	3	3	0	-1	3	-1	1	-1	0	-1	0	-1	1	0
χ_{13}	3	1	3	3	0	-1	3	1	-1	1	0	-1	0	-1	-1	0
χ_{14}	6	0	0	-6	0	-2	6	0	0	0	0	0	0	2	0	0
χ_{15}	12	-2	0	0	0	0	-4	0	0	2	0	0	0	0	0	0
χ_{16}	12	2	0	0	0	0	-4	0	0	-2	0	0	0	0	0	0
χ_1	2	0	0	0	2	2	2	A	0	0	0	0	0	0	A	0
χ_2	2	0	0	0	2	2	2	-A	0	0	0	0	0	0	-A	0
χ_3	4	0	0	0	-2	4	4	0	0	0	0	0	0	0	0	0
χ_4	6	0	0	0	0	-2	6	A	0	0	0	0	0	0	-A	0
χ_5	6	0	0	0	0	-2	6	-A	0	0	0	0	0	0	A	0
χ_6	12	0	0	0	0	0	-4	0	B	0	0	0	0	0	0	0
χ_7	12	0	0	0	0	0	-4	0	-B	0	0	0	0	0	0	0
χ_1	2	0	-2	2	-1	0	2	0	C	0	1	0	-1	0	-C	1
χ_2	2	0	-2	2	-1	0	2	0	-C	0	1	0	-1	0	C	1
χ_3	2	0	2	2	-1	0	2	0	C	0	-1	0	-1	0	C	-1
χ_4	2	0	2	2	-1	0	2	0	-C	0	-1	0	-1	0	-C	-1
χ_5	4	0	4	4	1	0	4	0	0	0	1	0	1	0	0	1
χ_6	4	0	-4	4	1	0	4	0	0	0	-1	0	1	0	0	-1
χ_7	4	0	0	-4	-2	0	4	0	0	0	0	0	2	0	0	0
χ_8	4	0	0	-4	1	0	4	0	0	0	D	0	-1	0	0	-D
χ_9	4	0	0	-4	1	0	4	0	0	0	-D	0	-1	0	0	D
χ_{10}	12	0	0	0	0	0	-4	-2	0	0	0	0	0	0	0	0
χ_{11}	12	0	0	0	0	0	-4	2	0	0	0	0	0	0	0	0
χ_1	4	0	0	0	-2	0	4	0	0	0	0	0	0	0	B	0
χ_2	4	0	0	0	-2	0	4	0	0	0	0	0	0	0	-B	0
χ_3	6	0	0	0	0	0	-2	-A	A	0	0	-2	0	-2	0	0
χ_4	6	0	0	0	0	0	-2	A	-A	0	0	-2	0	-2	0	0
χ_5	6	0	0	0	0	0	-2	A	A	0	0	2	0	-2	0	0
χ_6	6	0	0	0	0	0	-2	-A	-A	0	0	2	0	-2	0	0
χ_7	8	0	0	0	2	0	8	0	0	0	0	0	0	0	0	0
χ_8	12	0	0	0	0	0	-4	0	0	0	0	0	0	4	0	0

where $A = -\sqrt{2}$, $B = -2i$,
$C = -\sqrt{2}i$, $D = -\sqrt{3}i$

Table 10: $\mathrm{IrrProj}(G, \alpha_i)$ of $A_4 \times S_3$

$\alpha_1 = \alpha_2^2 = 1$

$[g]_G$	1a	2a	3a	2b	3b	6a	2c	3c	3d	6b	6c	3e
$\lvert C_G(g)\rvert$	72	24	18	24	36	6	8	18	9	12	6	9
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	1	-1	-1	1	1	1	-1	1
χ_3	1	-1	A	1	1	$-A$	-1	$\overline{A}$	A	1	$-\overline{A}$	$\overline{A}$
χ_4	1	-1	$\overline{A}$	1	1	$\overline{A}$	-1	A	$\overline{A}$	1	$-A$	A
χ_5	1	1	A	1	1	A	1	$\overline{A}$	A	1	$\overline{A}$	$\overline{A}$
χ_6	1	1	$\overline{A}$	1	1	$\overline{A}$	1	A	$\overline{A}$	1	A	A
χ_7	2	0	2	2	-1	0	0	2	-1	-1	0	-1
χ_8	2	0	B	2	-1	0	0	$\overline{B}$	$-A$	-1	0	$-\overline{A}$
χ_9	2	0	$\overline{B}$	2	-1	0	0	B	$-\overline{A}$	-1	0	$-A$
χ_{10}	3	-3	0	-1	3	0	1	0	0	-1	0	0
χ_{11}	3	3	0	-1	3	0	-1	0	0	-1	0	0
χ_{12}	6	0	0	-2	-3	0	0	0	0	1	0	0
χ_1	2	-2	-1	0	2	1	0	-1	-1	0	1	-1
χ_2	2	2	-1	0	2	-1	0	-1	-1	0	-1	-1
χ_3	2	-2	$-\overline{A}$	0	2	$\overline{A}$	0	$-A$	$-\overline{A}$	0	A	$-A$
χ_4	2	-2	$-A$	0	2	A	0	$-\overline{A}$	$-A$	0	$\overline{A}$	$-\overline{A}$
χ_5	2	2	$-\overline{A}$	0	2	$-\overline{A}$	0	$-A$	$-\overline{A}$	0	$-A$	$-A$
χ_6	2	2	$-A$	0	2	$-A$	0	$-\overline{A}$	$-A$	0	$-\overline{A}$	$-\overline{A}$
χ_7	4	0	-2	0	-2	0	0	-2	1	0	0	1
χ_8	4	0	$-\overline{B}$	0	-2	0	0	$-B$	$\overline{A}$	0	0	A
χ_9	4	0	$-B$	0	-2	0	0	$-\overline{B}$	A	0	0	$\overline{A}$

where $A = \frac{-1-\sqrt{3}i}{2}$, $B = -1-\sqrt{3}i$

Table 11: IrrProj(G,α_i) of 3^{1+2}:D_8
$\alpha_1 = \alpha_2^2 = 1$

$[g]_G$	1a	2a	2b	2c	3a	4a	6a	6b	6c	12a	12b	3b	3c
$\|C_G(g)\|$	216	12	12	24	18	12	6	6	12	12	12	18	108
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	1	1	-1	-1	1	1	1	1	1
χ_3	1	-1	1	1	1	-1	-1	1	1	-1	-1	1	1
χ_4	1	1	-1	1	1	-1	1	-1	1	-1	-1	1	1
χ_5	2	0	0	-2	2	0	0	0	-2	0	0	2	2
χ_6	4	-2	0	0	-2	0	1	0	0	0	0	1	4
χ_7	4	0	-2	0	1	0	0	1	0	0	0	-2	4
χ_8	4	0	2	0	1	0	0	-1	0	0	0	-2	4
χ_9	4	2	0	0	-2	0	-1	0	0	0	0	1	4
χ_{10}	6	0	0	-2	0	2	0	0	1	-1	-1	0	-3
χ_{11}	6	0	0	-2	0	-2	0	0	1	1	1	0	-3
χ_{12}	6	0	0	2	0	0	0	0	-1	-C	C	0	-3
χ_{13}	6	0	0	2	0	0	0	0	-1	C	-C	0	-3
χ_1	2	0	0	0	2	A	0	0	0	A	A	2	2
χ_2	2	0	0	0	2	-A	0	0	0	-A	-A	2	2
χ_3	4	0	0	0	-2	0	B	0	0	0	0	1	4
χ_4	4	0	0	0	-2	0	-B	0	0	0	0	1	4
χ_5	4	0	0	0	1	0	0	B	0	0	0	-2	4
χ_6	4	0	0	0	1	0	0	-B	0	0	0	-2	4
χ_7	6	0	0	0	0	A	0	0	C	D	E	0	-3
χ_8	6	0	0	0	0	A	0	0	-C	E	D	0	-3
χ_9	6	0	0	0	0	-A	0	0	C	-D	-E	0	-3
χ_{10}	6	0	0	0	0	-A	0	0	-C	-E	-D	0	-3

where A = -$\sqrt{2}$, B = -$\sqrt{3}i$, C = $\sqrt{3}$,
D = $E(24)^{17}$-$E(24)^{19}$, E = E(24)-$E(24)^{11}$

Table 12: IrrProj(G,α_i) for $S_4 \times S_3$

$$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$$

$[g]_G$	1a	6a	2a	3a	2b	3b	2c	4a	6b	2d	3c	6c	4b	2e	12a
$\lvert C_G(g)\rvert$	144	6	8	72	48	18	24	8	24	48	9	12	24	16	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	1	1	-1	1	1	-1	1	-1	-1	-1	-1
χ_3	1	1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	-1
χ_4	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1
χ_5	2	0	0	-1	2	2	-2	0	-1	0	-1	1	-2	0	1
χ_6	2	1	0	2	2	-1	0	0	2	-2	-1	0	0	-2	0
χ_7	2	-1	0	2	2	-1	0	0	2	2	-1	0	0	2	0
χ_8	2	0	0	-1	2	2	2	0	-1	0	-1	-1	2	0	-1
χ_9	3	0	1	3	-1	0	-1	-1	-1	-3	0	-1	1	1	1
χ_{10}	3	0	-1	3	-1	0	-1	1	-1	3	0	-1	1	-1	1
χ_{11}	3	0	-1	3	-1	0	1	1	-1	-3	0	1	-1	1	-1
χ_{12}	3	0	1	3	-1	0	1	-1	-1	3	0	1	-1	-1	-1
χ_{13}	4	0	0	-2	4	-2	0	0	-2	0	1	0	0	0	0
χ_{14}	6	0	0	-3	-2	0	-2	0	1	0	0	1	2	0	-1
χ_{15}	6	0	0	-3	-2	0	2	0	1	0	0	-1	-2	0	1
χ_1	4	0	0	4	0	-2	0	0	0	0	-2	0	0	0	0
χ_2	4	0	0	-2	0	-2	0	0	0	0	1	0	0	0	E
χ_3	4	0	0	-2	0	-2	0	0	0	0	1	0	0	0	-E
χ_4	4	A	0	4	0	1	0	0	0	0	1	0	0	0	0
χ_5	4	-A	0	4	0	1	0	0	0	0	1	0	0	0	0
χ_6	8	0	0	-4	0	2	0	0	0	0	-1	0	0	0	0
χ_1	2	0	0	2	2	2	0	0	2	0	2	0	0	0	0
χ_2	2	A	0	2	2	-1	0	0	2	0	-1	0	0	0	0
χ_3	2	-A	0	2	2	-1	0	0	2	0	-1	0	0	0	0
χ_4	2	0	0	-1	2	2	0	0	-1	0	-1	C	0	0	C
χ_5	2	0	0	-1	2	2	0	0	-1	0	-1	-C	0	0	-C
χ_6	4	0	0	-2	4	-2	0	0	-2	0	1	0	0	0	0
χ_7	6	0	0	6	-2	0	0	0	-2	0	0	0	0	0	0
χ_8	6	0	0	-3	-2	0	0	0	1	0	0	C	0	0	-C
χ_9	6	0	0	-3	-2	0	0	0	1	0	0	-C	0	0	C
χ_1	2	1	0	2	0	-1	0	B	0	-2	-1	0	-B	0	-B
χ_2	2	1	0	2	0	-1	0	-B	0	-2	-1	0	B	0	B
χ_3	2	-1	0	2	0	-1	0	-B	0	2	-1	0	-B	0	-B
χ_4	2	-1	0	2	0	-1	0	B	0	2	-1	0	B	0	B
χ_5	4	-1	0	4	0	1	0	0	0	-4	1	0	0	0	0
χ_6	4	1	0	4	0	1	0	0	0	4	1	0	0	0	0
χ_7	4	0	0	-2	0	-2	0	0	0	0	1	0	D	0	B
χ_8	4	0	0	-2	0	-2	0	0	0	0	1	0	-D	0	-B
χ_9	8	0	0	-4	0	2	0	0	0	0	-1	0	0	0	0

where A=$-\sqrt{3}i$, B = $\sqrt{2}i$, C = $\sqrt{3}$,
D = $-2\sqrt{2}i$, E =$-\sqrt{6}i$

In [17] it is mentioned that the sets $\mathrm{IrrProj}(G, \alpha_i)$ of G are only defined universally up to sign and it is possible that one can obtain different signs if the sets $\mathrm{IrrProj}(G, \alpha_i)$ are re-calculated using a different representation group $R = M(G).G$. But these signs are calculated consistently with a "special factor set" (as explained in Section 2) and so the inner product and conjugacy results of [4] apply to the sets $\mathrm{IrrProj}(G, \alpha_i)$.

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