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Finite Groups in which every Subgroup of Order Divisible by p is Normal

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Abstract

The article introduces two generalizations of Dedekind groups known as ${}_{pd}$Ddk-groups and ${}_{pd}$qHm-groups. In ${}_{pd}$Ddk-groups, subgroups with sizes that are multiples of a given prime p are normal, while in ${}_{pd}$qHm-groups, these subgroups are permutable. It is initially established that these groups belong to either p'-groups or supersolvable groups. It then proceeds to classify ${}_{pd}$Ddk-groups and ${}_{pd}$qHm-groups. The article concludes with a discussion of minimal non-${}_{pd}$Ddk-groups.

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1 Introduction

In this paper, our focus is solely on groups of finite order. Groups in which every subgroup is normal are termed Dedekind groups, and the non-abelian ones are referred to as Hamiltonian groups. The complete characterization of Dedekind groups and Hamiltonian groups was accomplished by R. Dedekind, E. Wendt, and R. Baer (see [1] and [4]).

Recall that two subgroups H and K are said to *permute* in G if

$$HK = KH.$$

A subgroup H is called permutable in G if H permutes with every subgroup of G. Besides, K. Iwasawa generalized the concept of Hamiltonian groups by looking at quasi-Hamiltonian groups, i.e. groups in which every subgroup is permutable and he classified all such finite groups in [8, Theorem 4].

Subsequent to this, numerous papers have delved into generalizations of this result. Two noteworthy contributions include the work of O.Y. Schmidt, who, in [12] and [13], was the first to study finite groups by imposing restrictions on their non-normal subgroups. He specifically examined the structure of finite groups where the non-normal subgroups either form a conjugacy class or constitute the union of two conjugacy classes.

E. Crestani [3] achieved a classification for all finite groups whose non-permutable subgroups have the same order and those that have a single conjugacy class of non-permutable subgroups.

In this article, we investigate two new extensions of Dedekind groups, referred to as ${}_{pd}$Ddk-groups and ${}_{pd}$qHm-groups. If p is a fixed prime number, in ${}_{pd}$Ddk-groups we specify that every subgroup whose order is divisible by p must be normal. Furthermore, in ${}_{pd}$qHm-groups, such subgroups are merely required to be permutable. At the beginning, it is established that these groups fall into either p'-groups or supersolvable groups. The article concludes with a classification of ${}_{pd}$Ddk-groups and ${}_{pd}$qHm-groups.

Before proceeding further, it is necessary to introduce some notation. Let n be a natural number; we denote by $\pi(n)$ the set of prime divisors of n. For a group G, we define $\pi(G) = \pi(|G|)$. Additionally, we say that a group G is a p'-group if $p \notin \pi(G)$.

Definition 1 Given a prime p, we call a subgroup H of a group G a pd-*subgroup* if $p \in \pi(H)$.

Definition 2 A group G is called ${}_{pd}$Ddk-*group* for p a prime if every pd-subgroup of G is normal in G.

Definition 3 A group G is called ${}_{pd}$qHm-*group* for p a prime if every pd-subgroup of G is permutable in G.

Remark 4 Every ${}_{pd}$Ddk-group is clearly a ${}_{pd}$qHm-group.

Remark 5 Every Dedekind group is clearly a ${}_{pd}$Ddk-group, and every quasi-Hamiltonian group is clearly a ${}_{pd}$qHm-group.

Remark 6 Every p′-group is a $_{pd}$Ddk-group and a $_{pd}$qHm-group. We will say that G is trivial $_{pd}$Ddk-group or trivial $_{pd}$qHm-group for such a group.

Example 7 For any prime p, p′-groups and Dedekind groups are examples of $_{pd}$Ddk-groups. Moreover, p′-groups, abelian group and quasi-Hamiltonian groups are examples of $_{pd}$qHm-groups.

Remark 8 A p-group need not be $_{pd}$Ddk-group or $_{pd}$qHm-group. Indeed, the dihedral group of order 8 is an example of 2-group that is not $_{2d}$Ddk-group.

The goal of this paper is to offer a classification of $_{pd}$Ddk-groups and $_{pd}$qHm-groups.

2 Auxiliary results

To initiate our exploration, we lay the groundwork by establishing foundational facts concerning $_{pd}$Ddk-groups and $_{pd}$qHm-groups.

Lemma 9 *If* G *is a* $_{pd}$*Ddk-group* (*respectively a* $_{pd}$*qHm-group*), *then* G *satisfies the following properties*:

(i) *every subgroup of* G *is* $_{pd}$*Ddk-group* (*respectively* $_{pd}$*qHm-group*). *In particular, the class of* $_{pd}$*Ddk-groups* (*respectively* $_{pd}$*qHm-groups*) *is closed for subgroups;*

(ii) *every factor group of* G *is* $_{pd}$*Ddk-group* (*respectively* $_{pd}$*qHm-group*). *In particular, the class of* $_{pd}$*Ddk-groups* (*respectively* $_{pd}$*qHm-groups*) *is closed for homomorphic images.*

Proof — (i) Let H be a subgroup of a $_{pd}$Ddk-group G and H_1 be a pd-subgroup in H. Then H_1 is pd-subgroup in G. Because G is $_{pd}$Ddk-group, H_1 must be normal in G. Therefore, H_1 is normal in H and hence H itself is a $_{pd}$Ddk-group.

(ii) Let N be a normal subgroup of a $_{pd}$Ddk-group G and let H be a pd-subgroup in G/N. Then, by the correspondence theorem, there exists a pd-subgroup H^* in G such that $H = H^*/N$. Since, by assumption, G is $_{pd}$Ddk-group, H^* must be normal in G. Therefore, H is normal in G and hence G/N itself is $_{pd}$Ddk-group.

The proof for $_{pd}$qHm-groups is basically identical to the proof above with replacing $_{pd}$Ddk-group by $_{pd}$qHm-group and normal subgroup by permutable subgroup. □

The direct product of two $_{pd}$Ddk-groups or $_{pd}$qHm-groups may not always result in a $_{pd}$Ddk-group or $_{pd}$qHm-group, as shown in the following example.

Example 10 Consider the group

$$G = \langle x, y : x^9 = y^3 = e, x^y = x^5 \rangle,$$

isomorphic to the semidirect product of a cyclic group of order 9 and a cyclic group of order 3. With an order of 27, G is a $_{2d}$Ddk-group. Moreover, the cyclic group $H = Z_2$ of order 2 is also a $_{2d}$Ddk-group. Now, by forming the group $D = G \times H$, the subgroup

$$S = \langle y \rangle \times H$$

emerges as a 2d-subgroup of D, yet it fails to be permutable within D.

3 Nontrivial $_{pd}$Ddk and $_{pd}$qHm-groups

In this section, we present results that establish significant properties of $_{pd}$Ddk-groups and $_{pd}$qHm-groups. This includes proving that nontrivial $_{pd}$Ddk-groups and $_{pd}$qHm-groups are supersolvable.

Theorem 11 *If* G *is a nontrivial* $_{pd}$*qHm-group, then* G *is supersolvable.*

Proof — Assume that G is a nontrivial $_{pd}$qHm-group, and let M be maximal subgroup of G.

Case 1: If $p \in \pi(M)$, then M is a permutable group and since it is also maximal by [11, 13.2.1], M is a normal subgroup that is also maximal, thus has prime index.

Case 2: If $p \notin \pi(M)$. Let $x \in G$ of order p and $N = \langle x \rangle$, since G is a nontrivial $_{pd}$Ddk-group N is permutable in G. Therefore M is a proper subgroup of $MN = G$ and thus the index of M in G is p.

So, based on both cases, any maximal subgroup M of G has prime index and hence, by Huppert's theorem [7], G is supersolvable. □

Corollary 12 *If* G *is nontrivial* ${}_{pd}$*Ddk-group, then* G *is supersolvable.*

Proof — Every ${}_{pd}$Ddk-group is a ${}_{pd}$qHm-group. □

Lemma 13 *If* $|G| = pq$ *where* p *and* q *are primes and* $p \geqslant q$*, then* G *is a* ${}_{pd}$*Ddk-group.*

Proof — If $p = q$, then G is abelian. Thus we may suppose $p > q$ in which case the Sylow p-subgroup of G is normal in G and the only other pd-subgroup is G itself. Hence, the result follows. □

Remark 14 The Dicyclic group of order 12 given by the presentation

$$\mathrm{Dic}_{12} = \left\langle a, x \mid a^6 = x^4 = 1,\ xax = a^{-1} \right\rangle$$

is a nontrivial non-Dedekind ${}_{pd}$Ddk-groups which is not of order pq.

Theorem 15 *If* G *is a nontrivial* ${}_{pd}$*Ddk-group with* $p = \min(\pi(G))$*, then* G *is a Dedekind group.*

Proof — Assume that G is a nontrivial ${}_{pd}$Ddk-group with

$$p = \min(\pi(G)),$$

and let H be a p'-subgroup of G, and $x \in G$ an element of order p. Let $K = \langle x \rangle H$. Note that $\langle x \rangle$ is normal in K and H having index

$$p = \min(\pi(K))$$

in K so is also normal by [10]. Therefore

$$K \simeq \langle x \rangle \times H$$

and H is characteristic in K, thus normal in G. Hence G is a Dedekind group. □

Remark 16 Note that, by Remark 8, not all supersolvable groups are ${}_{pd}$Ddk-groups. Also by Theorem 15 we see that, if $p \neq 2$ is a prime, then a nonabelian p-group is never a ${}_{pd}$Ddk-groups and note that such a group is nilpotent and thus supersolvable.

Theorem 17 *If* G *is a nontrivial* ${}_{pd}$*qHm-group with* $p = \min(\pi(G))$*, then* G *is a quasi-Hamiltonian group.*

Proof — Assume that G is a nonabelian nontrivial ${}_{pd}$qHm-group with $p = \min(\pi(G))$, and let H be a p'-subgroup of G. Now let J be a subgroup of G we will look at two cases.

Case 1: If $p \in \pi(J)$, then J is a permutable subgroup and $JH = HJ$.

Case 2: If $p \notin \pi(J)$, let $x \in G$ of order p and let $K = \langle x \rangle H$. Note that K is a subgroup since $\langle x \rangle$ is a permutable subgroup. Also note that since $\langle x \rangle$ is a permutable Sylow subgroup of K it is normal in K and H having index

$$p = \min(\pi(K))$$

in K is also normal by [10]. Therefore $K \simeq \langle x \rangle \times H$ and H is characteristic in K. Since $p \mid |K|$, K is permutable in G. Thus KJ is a subgroup of G. Using the same argument as above we see that

$$KJ = \langle x \rangle \times HJ.$$

Therefore HJ is a subgroup of G. So H is permutable in G. Hence G is a quasi-Hamiltonian group. □

4 Classification

In this section, we classify ${}_{pd}$Ddk-groups and ${}_{pd}$qHm-groups.

Corollary 18 *If* G *is a* ${}_{2d}$*qHm-group, then* G *is a Dedekind group or a* $2'$*-group.*

Proof — For a nontrivial ${}_{2d}$Ddk-group G, we observe that

$$2 = \min(\pi(G)),$$

and therefore, by Theorem 15, G is a Dedekind group. □

Corollary 19 *If* G *is a* ${}_{2d}$*Ddk-group, then* G *is a quasi-Hamiltonian group or a* $2'$*-group.*

Proof — Assume that G is a nontrivial ${}_{2d}$qHm-group and note that $2 = \min(\pi(G))$. Therefore, by Theorem 17, G is an abelian or a quasi-Hamiltonian group. □

Corollary 20 *If* G *is a* ${}_{2d}$*qHm-group, then* G *a solvable group.*

If G is a $2'$-group, then, by the Feit–Thompson Theorem [5], it is a solvable ${}_{2d}$qHm-group. Moreover, if $2 \in \pi(G)$, then G is solvable from Corollary 19.

Theorem 21 *Let* G *be a group. Then* G *is a* ${}_{pd}$*Ddk-group for* p *an odd prime if and only if* G *is isomorphic to one of the following groups*:

(1) *a* p'*-group,*

(2) *a Dedekind group,*

(3) $P \rtimes Q$ *where* P *is a cyclic subgroup of order* p *and* Q *is a* p'*-Dedekind group and* Q *acts as a group of nontrivial power automorphisms on* P*.*

Proof — Let G be a nontrivial ${}_{pd}$Ddk-group with p an odd prime. It is worth noting that the Sylow p-subgroup P of G is normal. We note that P is Dedekind, hence abelian. Since G is solvable, according to Theorem 11, we can deduce from the Schur–Zassenhaus Theorem (see [11, 9.1.2]) that G can be expressed as the semidirect product

$$G = P \rtimes Q,$$

where Q is a Hall p'-complement of P. Additionally, observe that all complements of P in G are conjugate. Also note that since $Q \simeq G/P$ it must be a Dedekind group. Lets look at two cases.

Case 1: If P is not cyclic, then there exist elements x and y in P of order p, and $\langle x\rangle \cap \langle y\rangle = \{1\}$. Let H be a subgroup of G. We can assume that H is a p'-group, as otherwise, it is normal by definition. Furthermore, by definition, $\langle x\rangle H$ and $\langle y\rangle H$ are both normal in G. Consequently, $H = \langle x\rangle H \cap \langle y\rangle H$ is normal in G. Thus, G is a Dedekind group.

Case 2: If $P = \langle x\rangle$ is cyclic, then G can be represented as

$$G = P \rtimes Q,$$

where Q is a p'-Dedekind group. It is important to note that Q acts as power automorphisms on P since P is cyclic. Now assume

$$|P| = p^n > p,$$

and let $z \in P$ be an element of order p^2 and $r \in Q$ an element of prime order that acts nontrivially on x. Note that r will also act nontrivially

on z. Note that class of r in $P\langle r\rangle$ is

$$Cl_{P\langle r\rangle}(r) = \{r, xr, \dots, x^{p^n-1}r\} \subset Cl_G(r)$$

which is not a subset of $H = \langle z^p\rangle\langle r\rangle$. Thus $\langle r\rangle$ is not normal, a contradiction.

Now assume that

$$G = P \rtimes Q$$

where $P = \langle x\rangle$ is a cyclic subgroup of order p and Q is a p'-Dedekind group and Q acts as a nontrivial power automorphisms on P. Let

$$P \neq H \leqslant G$$

with $p \in \pi(H)$, since P is the unique subgroup of order p, $P \leqslant H$ and H/P is a subgroup of G/P which is Dedekind, thus H/P is normal in G/P. Therefore H is normal in G. □

Remark 22 A T-group, as defined, is a group where the property of normality is transitive. In 1957, W. Gaschütz provided a description of soluble T-groups, see [6]. These groups, denoted by G, have a nilpotent residual that is an abelian Hall subgroup L of odd order, satisfying the conditions that G/L is Dedekind and G normalizes every subgroup of L. Note also that, Gaschütz's results, along with Corollary 19 and Theorem 21, lead to the conclusion in Corollary 23.

Corollary 23 *If* G *is a nontrivial* $_{pd}$*Ddk-group, then* G *is a* T*-group.*

Remark 24 A PT-group, by definition, is a group where the property of permutability is transitive. In 1964, G. Zacher provided a description of soluble PT-groups, see [2] and [15]. These groups, denoted by G, have a nilpotent residual that is an abelian Hall subgroup L of odd order, satisfying the conditions that G/L is abelian or a quasi-Hamiltonian and G normalizes every subgroup of L.

Lemma 25 *Let* G *be a group. If* G *is a nontrivial, nonabelian and non-quasi-Hamiltonian* $_{pd}$*qHm-group with* p *an odd prime, then*

$$G = P \rtimes Q$$

where P *is an abelian* p*-group and* Q *is an abelian or a quasi-Hamiltonian* p'*-group which acts as a nontrivial power automorphisms on* P*. In particular* G *is a metanilpotent supersovable group.*

Proof — Consider a nontrivial, nonabelian, and nonquasi-Hamiltonian $_{pd}$qHm-group G, where p is an odd prime. Hence, the Sylow p-subgroup P of G is normal because it is a permutable Sylow subgroup. Since G is solvable according to Theorem 11, we can deduce from the Schur–Zassenhaus Theorem (see [11, 9.1.2]) that G can be expressed as the semidirect product

$$G = P \rtimes Q,$$

where Q is a Hall p'-complement of P. Additionally, observe that all complements of P in G are conjugate.

Besides, note that P is a $_{pd}$qHm-group, making it a quasi-Hamiltonian p-group. Moreover, it is important to observe that by correspondence theorem, Q is also a quasi-Hamiltonian or an abelian p'-group. Consequently, G is metanilpotent, and by Theorem 11, it is also supersolvable. Now, let $x \in P$ and H a complement of P in G. Note that $\langle x \rangle$ is a permutable subgroup and

$$\big(|\langle x \rangle|, |H|\big) = 1,$$

thus, by [11],

$$H \leqslant N_G\big(\langle x \rangle\big).$$

Therefore Q acts as a nontrivial power automorphisms on P.

Note that, by [8] and [14], if P is not abelian, then

$$P = \langle x, A \rangle$$

where A is an abelian p-group $\langle x \rangle$ is a cyclic p-group and

$$a^x = a^{1+p^r},$$

for any $a \in A$. Thus the normal subgroups of P are P and all the subgroups of A. Now, since Q acts as power automorphisms on P, all the normal subgroups of P are normal in G. Thus, by [2, Lemma 1], it follows that G is a PT-group Therefore, by [15], P is abelian. □

Corollary 26 *If* G *is a nontrivial* $_{pd}$*qHm-group, then* G *is a* PT*-group.*

Proof — Zacher's results [2] and [15], along with Corollary 18 and Lemma 25 prove the claim. □

Theorem 27 *Let* G *be a group. Then* G *is a* ${}_{pd}qHm$*-group, for* p *an odd prime, if and only if* G *is isomorphic to one of the following groups*:

(1) *a* p'*-group,*

(2) *a quasi-Hamiltonian group,*

(3) $P \rtimes Q$ *where* P *is a cyclic* p*-group of order* p *and* Q *is an abelian or a quasi-Hamiltonian* p'*-group which acts as a nontrivial power automorphisms on* P*.*

Proof — Consider a nontrivial, nonabelian, and nonquasi-Hamiltonian ${}_{pd}qHm$-group G, where p is an odd prime. By Lemma 25,

$$G = P \rtimes Q$$

where P is an abelian p-group and Q is an abelian or a quasi-Hamiltonian p'-group which acts as a nontrivial group of power automorphisms on P.

Assume that P is not cyclic, then there exist elements x and y in P of order p, and $\langle x\rangle \cap \langle y\rangle = \{1\}$. Let H be a subgroup of G. We can assume that H is a p'-group, as otherwise, it is permutable by definition. Furthermore, by definition, $\langle x\rangle H$ and $\langle y\rangle H$ are both permutable in G. Let

$$K = \langle x\rangle\langle y\rangle H.$$

Since $\langle x\rangle H$ and $\langle y\rangle H$ are maximal permutable subgroups of K they are normal in K. Consequently,

$$H = \langle x\rangle H \cap \langle y\rangle H$$

is normal in K. Thus, since G is a PT-group, H is permutable in G and G is quasi-Hamiltonian.

Now assume that $P = \langle x\rangle$ is cyclic, $|P| = p^n > p$, and let $z \in P$ be an element of order p^{n-1} and $r \in Q$ an element of prime order that acts nontrivially on x. Note that r will also act nontrivially on z. The subgroup

$$H = \langle z\rangle\langle r\rangle$$

is permutable in G, thus it is permutable and maximal in $P\langle r\rangle$, i.e. it is normal in $P\langle r\rangle$.

Note that class of r is

$$Cl_{P\langle r\rangle}(r) = \left\{r, xr, \ldots, x^{p^n-1}r\right\}$$

is not a subset of $H = \langle z \rangle \langle r \rangle$, thus it is not normal, a contradiction.

Now assume that $G = P \rtimes Q$, where $P = \langle x \rangle$ is a cyclic subgroup of order p and Q is a quasi-Hamiltonian p'-group which acts as a nontrivial power automorphisms on P. Let

$$P \neq H \leqslant G$$

with $p \in \pi(H)$. Since P is the unique subgroup of order p, $P \leqslant H$. Moreover, H/P is a subgroup of G/P which is a quasi-Hamiltonian, thus H/P is permutable in G/P. Therefore H is permutable in G. □

Remark 28 Observe that, while it is true that every ${}_{pd}$Ddk-group is a ${}_{pd}$qHm-group (every T-group is a PT-group), the converse is not necessarily true. For example, any quasi-Hamiltonian p-group, for an odd prime p, is a ${}_{pd}$qHm-group and a PT-group, but it is not a ${}_{pd}$Ddk-group or a T-group.

It is also important to note that one can find PT-groups with a non-cyclic Hall subgroup of odd order divisible by a prime p. Such a group will not be a ${}_{pd}$qHm-group according to Theorem 27. Additionally, consider the Suzuki group $Sz(q)$, which has order

$$q^2(q-1)(q^2+1)$$

and is not divisible by 3. Therefore, it is a ${}_{3d}$qHm but not a PT-group.

5 Minimal non ${}_{pd}$Dedekind-groups

In this section, we introduce the critical $\mathfrak{X}$-groups where $\mathfrak{X}$ is the class of all finite ${}_{pd}$Ddk-groups. In particular, we prove that a critical ${}_{2d}$Ddk-groups must be solvable. Let us initially recall the following definitions.

Definition 29 Given a class of groups $\mathfrak{X}$, a group G is called a *minimal non* $\mathfrak{X}$-group, or an $\mathfrak{X}$-*critical* group, if G is not in $\mathfrak{X}$ but all its proper subgroups are in $\mathfrak{X}$.

Definition 30 A group G is called a *minimal non* ${}_{pd}$*Ddk*-group, if G is not ${}_{pd}$Ddk-group but all its proper subgroups are ${}_{pd}$Ddk-groups.

Example 31 The alternating group A_5 is an example of minimal non ${}_{5d}$Ddk-group, since the 5d-subgroups have orders 5 and 10, and they are ${}_{5d}$Ddk-groups. Note that A_5 is simple.

Example 32 The alternating group A_4 is an example of minimal non $_{2d}$Ddk-group. Note that A_4 is solvable.

Theorem 33 *If* G *is a minimal non* $_{2d}$*Ddk-group, then* G *must be a solvable group.*

Proof — Note that a minimal non $_{2d}$Ddk-group must have even order. Let G be a minimal non $_{2d}$Ddk-group. If G has a non trivial normal subgroup N, then N is solvable by Corollary 20 and G/N is either minimal non $_{2d}$Ddk-group or a $_{2d}$Ddk-group.

Suppose, for the sake of contradiction, that there exists a nonsolvable minimal non-$_{2d}$Ddk-group G. From the previous discussion, we can assume that G is simple. Therefore, by Corollary 18, the normalizer of its Sylow 2-subgroup must be a Dedekind group, which contradicts the result in [9]. □

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