# Giovanni Zacher (1926 – 2021)

Giovanni Zacher was born in Bolzano on October 25, 1926, in a German speaking family. His father Artur Zacher was a high rank officer in the city police of Bolzano; his mother Edwige Baur came from a family of landowners and wine merchants. Giovanni had an older brother Richard who died in a car accident in 1945; and a twin sister Anne Marie who still lives in their family house in Bolzano.

Zacher's education was slightly non standard: he followed the five years curriculum of *ginnasio* at the school in Bolzano held by the Franciscan friars, then one year at the state Liceo classico in Bolzano. At this point (1943) his family thought it better to leave Bolzano for the Val di Ledro where there was a facility of the Seminar under the rule of the bishop of Trento; and it was here that he completed his *liceo classico*. At the end of the war he entered the university of Padua as a student of mathematics and physics. For his graduation in 1950 he chose a thesis on "Groups and Galois Theory", his advisor being Giuseppe Scorza Dragoni, a mathematical analyst with more than an interest in algebra. Giuseppe Scorza Dragoni's father Gaetano Scorza, who was an important exponent of the Rome school of algebraic geometry, is also the author of the first Italian book on group theory: his "Gruppi Astratti" was posthumously published in 1942 edited by Giuseppe Scorza Dragoni and Guido Zappa. And in 1951 Zacher is in Napoli with a one year scholarship to work with Zappa who was currently studying homomorphisms between subgroup lattices.

This is certainly the origin of Zacher's first three published papers (1951). In the same year he was appointed assistant of mathematical analysis at the university of Padua, and for several years gave analysis courses. But it was already clear that not analysis but algebra, and actually group theory, was his favourite research interest. He was deeply impressed by Reinhold Baer's long paper (1939) "The significance of the system of subgroups for the structure of the group". A particular subject caught his attention: groups with dual (the group G has a *dual* $\widehat{G}$ if there is an inclusion reversing bijection of the subgroup lattice of G onto the subgroup lattice of $\widehat{G}$). The subject had been introduced by Baer in 1937, the finite soluble groups with dual were determined by Michio Suzuki in 1951. In a series of papers (1960–61) Zacher extended these results to arbitrary soluble groups, and showed that finite groups with dual are soluble. He also came back to this topic in 1966 and 1971, when he was able to handle the locally finite case — but this required the Odd Order Theorem! — and again (with S.E. Stonehewer) in 1995, 1996 and (with M. Costantini and S.E. Stonehewer) in 2001.

Zacher spent the year 1960 at the University of Illinois at Urbana Champaign, where he met Michio Suzuki, the author of the Springer text "Structure of a group and the structure of its lattice of subgroups" – this was the time when Suzuki introduced the family of simple groups that now bears his name. Many years later (1994) Suzuki came to Padova as visiting professor.

In 1961 Zacher won the first *concorso* in Italy for a chair in Algebra (the triple of winners being Zacher, Curzio, Trevisan) and became professor of Algebra in Padova, where he was going to spend almost his whole career. The exception are the years 1978–81 when he moved to Trento and helped to establish the new-born Faculty of Sciences of the local university; but in 1981 he was back in Padova as professor of *Istituzioni di Algebra Superiore*.

Zacher gave an important contribution to the university of Padova and its mathematical community, first grouped in the *Seminario Matematico*, and later in the *Dipartimento di Matematica*. Many former students still have a vivid memory of his monographic courses on special aspects of algebra related to group theory, the source of useful lecture notes and research problems at various levels. He always had some visitors from abroad, and was eager to put them in contact with colleagues and students: this helped to give the mathematical community a lively atmosphere, and was the starting point of several

long-lasting scientific collaborations (in particular with Kiel, Urbana, Freiburg, Warwick, Michigan State).

From 1964 to 1993 he was editor in chief of the journal *Rendiconti del Seminario Matematico dell'Università di Padova*, that under his direction came to publish about 250 pages of original research papers each year. Thanks to an appropriate exchanges policy, the journal was also an important asset for the library. In short, one of the best mathematical journals in Italy, and a library with an excellent coverage of current journals — rules changed with the rise of e-publishing and digitalization, and with the growth of commercial publishers at the expenses of academic ones.

Giovanni Zacher is the author of more than 65 papers in group theory, almost all of them dealing with questions in which the lattice of subgroups is involved. He touched on many different aspects in this area; it would be a difficult task to mention all of them, so I will only report on two research lines that appear to have interested him along many years. One such line is "complements in subgroup lattices": Zacher's first contribution is the characterization of finite groups whose subgroup lattice is relatively complemented (1952), then he describes the finite soluble groups with complemented subgroup lattice (1953); a characterization of finite groups with complemented subgroup lattice is still missing, but finite simple groups have this property (with M. Costantini, 2004). "Projectivities" is the most frequented topic in Zacher's scientific production — a projectivity of G to the group $\widehat{G}$ is an isomorphism of their respective lattices of subgroups. In this subject a key result is Zacher–Rips' theorem (1980) that projectivities map finite index subgroups to finite index subgroups of the image; it is one of the tools involved in Zacher's deep results on the images of normal subgroups under projectivities (1982, with F. Napolitani 1983, 1985, 1999). The subgroup index problem is again investigated in Zacher's last paper (with M. Costantini, 2011) when he was 85!

But perhaps a result that made him equally proud is the creation of a rich school based in Padova, in which he first recruited some young research assistants looking for interesting problems and careful guidance, then students working for a thesis under his direction, or coming to Padova with a scholarship, and then students of students: now arriving the fourth generation. At the start, subgroup lattices were of course important! This faded in the long run, as new aspects of group theory came to the front and different areas of math-

ematics were investigated. Still, a number of people, when attacking their chosen new research question, give a grateful thought to Giovanni Zacher.

*Federico Menegazzo*