Fischer Matrices of a Factor Group of a Maximal Subgroup of the Baby Monster

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Outline

Introduction

- The Baby Monster $\mathbb{B}$ contains a maximal subgroup of structure $MS = 2^{9+16} \cdot Sp_8(2)$ that is a non-split extension of a special 2-group $P = 2^{9+16}$.
- The ordinary character table of MS has been computed with MAGMA and its character table is stored in the GAP library.

Introduction

- MS has two elementary abelian 2-subgroups $E_1 = 2^8$ and $E_2 = 2^9$ such that $E_1, E_2 \trianglelefteq MS$.
- Hence the factor groups $\frac{MS}{E_1} \cong 2_+^{1+16 \cdot} Sp_8(2)$ and $\frac{MS}{E_2} \cong \overline{G_1} = 2^{16 \cdot} Sp_8(2)$ exist.
- In this talk, we show how the Fischer matrices and ordinary character table of the factor group $\overline{G} = \frac{MS}{E_1} \cong 2_+^{1+16 \cdot} Sp_8(2)$ can be constructed from the Fischer matrices of $\frac{MS}{E_2} \cong \overline{G_1} = 2^{16 \cdot} Sp_8(2)$ using a technique called `lifting of Fischer matrices`[5],

The conjugacy classes of MS, $\overline{G}$, $\overline{G_1}$ and G

Figure 1 represents the commutative diagram which is associated with the natural homomorphisms $\eta_1{:}MS \to \overline{G}$, $\eta_2{:}\overline{G} \to \overline{G_1}$, $\eta_3{:}\overline{G_1} \to G$, $\eta{:}MS \to G$ and $\eta_4{:}\overline{G} \to G$. The kernels of the extensions are $ker(\eta_1) = E_1$, $ker(\eta_2) = Z(\overline{G})$, $ker(\eta_3) = E_3 \cong 2^{16}$, $ker(\eta_4 = \eta_3 \circ \eta_2) = E_3$ and $ker(\eta = \eta_4 \circ \eta_1) = 2^{9+16}$.

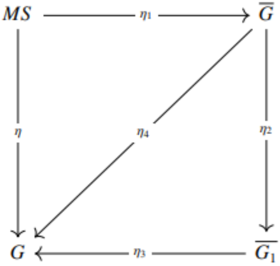

Figure 1. Commutative diagram

The conjugacy classes of MS, $\overline{G}$, $\overline{G_1}$ and G

Let's consider $\overline{G_1}$, then G is identified with $\frac{\overline{G_1}}{E_3}$ under the map η_3. In addition, the pre-image $\eta_3^{-1}([E_3\overline{q}])$ of a conjugacy class $[E_3\overline{q}]$ in $\frac{\overline{G_1}}{E_3}$ is a union $\bigcup_{i=1}^{\widehat{c(g)}}[\overline{q_i}]$ of say $\widehat{c(g)}$ conjugacy classes $[\overline{q_i}]$ in $\overline{G_1}$. Note that a coset $E_3\overline{q}$ is identified with a $g \in G$ such that $\overline{q}$ is a lifting for g to $\overline{G}$. Therefore, above a coset representative $E_3\overline{q} \in [E_3\overline{q}]$ lies a set $\widehat{X(g)} = \{\overline{q_1} = \overline{q}, \overline{q_2}, \ldots, \overline{q_{\widehat{c(g)}}}\}$ of representatives of classes $[\overline{q_i}]$ of $\overline{G_1}$. Similarly, a pre-image $\eta_1^{-1}([\overline{q_i}])$ of a class $[\overline{q_i}]$ of $\overline{G_1}$, $\overline{q_i} \in \widehat{X(g)}$, is a union $\bigcup_{j=1}^{c(\overline{q_i})}[\overline{g_{i_j}}]$ of $c(\overline{q_i})$ classes $[\overline{g_{i_j}}]$ of $\overline{G}$. Note that $\overline{q_i} \in \widehat{X(g)}$ is identified with a coset $Z(\overline{G})\overline{g_i} \in \frac{\overline{G}}{Z(\overline{G})} \cong \overline{G_1}$ where $\overline{g_i}$ is a lifting for $\overline{q_i}$ in $\overline{G}$. Hence a set $X(\overline{q_i}) = \{\overline{g_{i_1}} = \overline{g_i}, \overline{g_{i_2}}, \ldots, \overline{g_{i_{c(\overline{q_i})}}}\}$ of representatives of conjugacy classes $[\overline{g_{i_j}}]$ is obtained from the coset $Z(\overline{G})\overline{g_i}$. Since $\eta_4 = \eta_3 \circ \eta_2$ (see Figure 1), it follows that the pre-image $\eta_4^{-1}(g)$ for $g \in G$ is the set

$$\overline{X(g)} = \bigcup_{i=1}^{\widehat{c(g)}} X(\overline{q_i}) = X(\overline{q_1}) \bigcup X(\overline{q_2}) \bigcup \cdots \bigcup X(\overline{q_{\widehat{c(g)}}})$$

The conjugacy classes of MS, $\overline{G}$, $\overline{G_1}$ and G

Following the trend above, a pre-image $\eta_1^{-1}([\overline{g_{i_j}}])$ of a class $[\overline{g_{i_j}}]$ of $\overline{G}$, $\overline{g_{i_j}} \in \overline{X(g)}$, is a union $\bigcup_{z=1}^{c(\overline{g_{i_j}})}[\overline{m_{i_{j_z}}}]$ of $c(\overline{g_{i_j}})$ classes $[\overline{m_{i_{j_z}}}]$ of $\overline{MS}$. Also, $\overline{g_{i_j}} \in \overline{X(g)}$ is identified with a coset $E_1\overline{m_{i_j}} \in \frac{\overline{MS}}{E_1} \cong \overline{G}$ where $\overline{m_{i_j}}$ is a lifting for $\overline{g_{i_j}}$ in $\overline{M}$. Hence a set $X(\overline{g_{i_j}}) = \{\overline{m_{i_{j_1}}} = \overline{m_{i_j}}, \overline{m_{i_{j_2}}}, \ldots, \overline{m_{i_{j_{c(\overline{g_{i_j}})}}}}\}$ of representatives of conjugacy classes $[\overline{m_{i_{j_z}}}]$ is obtained from the coset $E_1\overline{m_{i_j}}$.

The conjugacy classes of $\overline{G_1}$

- A 16-dimensional matrix group over $GF(2)$ which acts absolutely irreducibly on its unique module 2^{16} is available via the online ATLAS [7].
- Therefore, a split extension of shape $\overline{S} \cong E_3{:}G$ exists.
- The Fischer matrices and ordinary character table of $\overline{S}$ was constructed in [4].
- $\overline{S}$ has four orbits of lengths 1, 32640, 30600 and 2295 on $\mathrm{Irr}(E_3)$ with corresponding inertia factor groups $H_1 = Sp_8(2)$, $H_2 = Sp_6(2)$, $H_3 = 2^7{:}G_2(2)$ and $H_4 = 2^{10}{:}A_8$.

The conjugacy classes of $\overline{G}$ and $\overline{G_1}$

- With the aid of GAP and the character table of MS (uploaded in the GAP library) it can be seen that $\overline{G_1}$ has also four orbits of lengths 1, 32640, 30600 and 2295 on $\mathrm{Irr}(E_3)$ and that only the ordinary irreducible characters of the inertia factors are involved in the character table of $\overline{G}$.
- This means that the irreducible characters of the split extension $\overline{G}$ are the same as the ones for the non-split extension $\overline{G_1}$.
- The conjugacy classes $[x]_{\overline{G}}$ of $\overline{G}$ are obtained from the conjugacy classes $[g]_G$ of $G = Sp_8(2)$ by the method of *coset analysis* and are listed in Table 1 below.
- This method is well documented in [2, 3].

The conjugacy classes of $\overline{G}$ and $\overline{G_1}$

G has 5 orbits on $P_1 = \frac{P}{E_1} \cong 2_+^{1+16}$ of lengths 1, 1, 65280, 61200 and 4590. The first class contains the identity element, the second class the central element of order two, the third class $2^{16} - 2^8 = 65280$ elements of order 4 the fourth class 61200 elements of order two and fifth class 4590 elemnts of order 2.

Conjugacy Classes of $\overline{G}$ and and $\overline{G_1}$

Table 1. Conjugacy Classes of $\overline{G}$ and $\overline{G_1}$

$[g]_G$	k	f_j	m_j	$[x]_{\overline{G_1}}$	$\lvert C_{\overline{G_1}}(x)\rvert$	$\lvert C_{\overline{G}}(x)\rvert$	$[x]_{\overline{G}}$	$[g]_G$	k	f_j	m_j	$[x]_{\overline{G}}$	$\lvert C_{\overline{G_1}}(x)\rvert$	$\lvert C_{\overline{G}}(x)\rvert$	$[x]_{\overline{G}}$
1A	65536	1	1	1A	3104939232460800	6209878464921600	1A	2A	4096	1	16	2D	36238786560	72477573120	2D
						6209878464921600	2A							72477573120	2E
		2295	2295	2A	1352914698240	1352914698240	2B			45	720	2E	805306368	805306368	2F
		30600	30600	2B	101468602368	101468602368	2C			90	1440	2F	402653184	402653184	2G
		32640	32640	2C	95126814720	95126814720	4A			120	1920	2G	301989888	301989888	4B
										960	15360	4A	37748736	37748736	4C
										2880	46080	4B	12582912	12582912	4D

Action of $\overline{G}$ on $\mathrm{Irr}(P_1)$

G has five orbits of lengths 1, 32640, 30600 and 2295 on $\mathrm{Irr}(E_3)$ with corresponding inertia factor groups $H_1 = Sp_8(2) \cong H_2$, $H_3 = Sp_6(2)$, $H_4 = 2^7{:}G_2(2)$ and $H_5 = 2^{10}{:}A_8$.

Definition of the Fischer-Clifford Matrix

For a group extension $\overline{G} = P.G$, where P is a p-group and $g \in G$ is a conjugacy class representative, the Fischer-Clifford matrix $M(g) = \left(a^j_{(i,y_k)}\right)$ is defined as (see [1]):

$$a^j_{(i,y_k)} = \sum_{s=1}^{c(k)} \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H}_i}(y_{k_s})|}\psi_i(y_{k_s}),$$

where $x_j \in X(g)$ are class representatives in the coset $P\overline{g}$, $\overline{g} \in \overline{G}$ a lifting for $g \in G$ and $\psi_i \in \mathrm{Irr}(\overline{H}_i)$ extends $\theta_i \in \mathrm{Irr}(P)$.

Explanation of Terms in the Definition

- $R(g) = \{(i, y_k) \mid 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\}$
- t: number of orbits of G on $\mathrm{Irr}(P)$
- y_k: representatives of conjugacy classes in H_i fusing into $[g]$ in G
- $X(g) = \{x_1, \ldots, x_{c(g)}\}$: class representatives in $\overline{G}$ from coset $P\overline{g}$
- $\eta^{-1}(y_k) = \bigcup_{s=1}^{c(k)} \{y_{k_s}\}$: pre-images of $y_k \in H_i$ under $\eta : \overline{H}_i \to H_i$
- $\overline{H}_i = P.H_i$: inertia group of $\theta_i \in \mathrm{Irr}(P)$

Arithmetic Properties of $M(g)$

In practice, computing the elements y_{k_s} or the character tables of $\overline{H}_i$ is difficult because:

- The sets $\mathrm{Irr}(\overline{H}_i)$ are often large and complex.
- The character table of $\overline{G}$ is typically easier to compute than those of the inertia groups.

Instead of using the formal definition, the arithmetic properties of $M(g)$ are used to compute its entries (see [3]).

Arithmetic Properties of $M(g)$

1. $a^{i}_{(1,g)} = 1$ for all $i = \{1, 2, .., \widehat{c(g)}\}$.
2. $|\widehat{X(g)}| = |R(g)|$.
3. $\sum_{i=1}^{\widehat{c(g)}} m_i \, a^{i}_{(s,y_k)} \, \overline{a^{i}_{(s',y'_k)}} = \delta_{(s,y_k),(s',y'_k)} \frac{|C_G(g)|}{|C_{H_s}(y_k)|} \, |E_3|$.
4. $\sum_{(s,y_k)\in R(g)} a^{i}_{(s,y_k)} \, \overline{a^{i'}_{(s,y_k)}} \, |C_{H_s}(y_k)| = \delta_{ii'} \, |C_{\overline{G_1}}(\overline{q_i})|$.
   Since E_3 is an elementary abelian 2-group, the relations 5-7 below, hold as well for $\widehat{M(g)}$.
5. $a^{1}_{(s,y_k)} = \frac{|C_G(g)|}{|C_{H_s}(y_k)|}$.
6. $|a^{1}_{(s,y_k)}| \geq |a^{i}_{(s,y_k)}|$.
7. $a^{i}_{(s,y_k)} \equiv a^{1}_{(s,y_k)} (\text{mod } 2)$.

Structure of the Fischer-Clifford Matrix of $\overline{G_1}$

$$
\begin{array}{c}
\\
|C_G(g)| \\
|C_{H_2}(y_1)| \\
|C_{H_2}(y_2)| \\
\vdots \\
|C_{H_3}(y_1)| \\
|C_{H_3}(y_2)| \\
\vdots \\
|C_{H_4}(y_1)| \\
|C_{H_4}(y_2)| \\
\vdots
\end{array}
\left(
\begin{array}{cccc}
|C_{\overline{F}}(\overline{q_1})| & |C_{\overline{F}}(\overline{q_2})| & \cdots & |C_{\overline{F}}(\overline{q_{\widehat{c(g)}}})| \\
1 & 1 & \cdots & 1 \\
\hline
\frac{|C_G(g)|}{|C_{H_2}(y_1)|} & a^2_{(2,y_1)} & \cdots & a^{\widehat{c(g)}}_{(2,y_1)} \\
\frac{|C_G(g)|}{|C_{H_2}(y_2)|} & a^2_{(2,y_2)} & \cdots & a^{\widehat{c(g)}}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
\frac{|C_G(g)|}{|C_{H_3}(y_1)|} & a^2_{(3,y_1)} & \cdots & a^{\widehat{c(g)}}_{(3,y_1)} \\
\frac{|C_G(g)|}{|C_{H_3}(y_2)|} & a^2_{(3,y_2)} & \cdots & a^{\widehat{c(g)}}_{(3,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
\frac{|C_G(g)|}{|C_{H_4}(y_1)|} & a^2_{(4,y_1)} & \cdots & a^{\widehat{c(g)}}_{(4,y_1)} \\
\frac{|C_G(g)|}{|C_{H_4}(y_2)|} & a^2_{(4,y_2)} & \cdots & a^{\widehat{c(g)}}_{(4,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
m_1 & m_2 & \cdots & m_{\widehat{c(g)}}
\end{array}
\right)
$$

Figure: The Fischer-Clifford Matrix $\widehat{M(g)}$

Structure of the Fischer-Clifford Matrix of $\overline{G}$

$$
\begin{array}{c|ccccc}
 & & X(\overline{q_1}) & X(\overline{q_2}) & & X(\overline{q_{\widehat{c(g)}}}) \\
 & |C_{\overline{G}}(\overline{g_{1_1}})| & |C_{\overline{G}}(\overline{g_{1_2}})| & |C_{\overline{G}}(\overline{g_{2_1}})| & \cdots & |C_{\overline{G}}(\overline{g_{\widehat{c(g)}_1}})| \\
\hline
|C_G(g)| & 1 & 1 & 1 & \cdots & 1 \\
\hline
|C_{H_2}(y_1)| & \frac{|C_G(g)|}{|C_{H_2}(y_1)|} & \frac{|C_G(g)|}{|C_{H_2}(y_1)|} & a^{2_1}_{(2,y_1)} & \cdots & a^{\widehat{c(g)}_1}_{(2,y_1)} \\
|C_{H_2}(y_2)| & \frac{|C_G(g)|}{|C_{H_2}(y_2)|} & \frac{|C_G(g)|}{|C_{H_2}(y_2)|} & a^{2_1}_{(2,y_2)} & \cdots & a^{\widehat{c(g)}_1}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
|C_{H_3}(y_1)| & \frac{|C_G(g)|}{|C_{H_3}(y_1)|} & \frac{|C_G(g)|}{|C_{H_3}(y_1)|} & a^{2_1}_{(3,y_1)} & \cdots & a^{\widehat{c(g)}_1}_{(3,y_1)} \\
|C_{H_3}(y_2)| & \frac{|C_G(g)|}{|C_{H_3}(y_2)|} & \frac{|C_G(g)|}{|C_{H_3}(y_2)|} & a^{2_1}_{(3,y_2)} & \cdots & a^{\widehat{c(g)}_1}_{(3,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
|C_{H_4}(y_1)| & \frac{|C_G(g)|}{|C_{H_4}(y_1)|} & \frac{|C_G(g)|}{|C_{H_4}(y_1)|} & a^{2_1}_{(4,y_1)} & \cdots & a^{\widehat{c(g)}_1}_{(4,y_1)} \\
|C_{H_4}(y_2)| & \frac{|C_G(g)|}{|C_{H_4}(y_2)|} & \frac{|C_G(g)|}{|C_{H_s}(y_2)|} & a^{2_1}_{(4,y_2)} & \cdots & a^{\widehat{c(g)}_1}_{(4,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
|C_{H_5}(y_1)| & a^{1_1}_{(5,y_1)} & -a^{1_1}_{(5,y_1)} & 0 & \cdots & 0 \\
\hline
 & m_{1_1} & m_{1_2} & m_{2_1} & \cdots & m_{\widehat{c(g)}_1}
\end{array}
$$

Entries of last row of M(g)

Theorem

The row of a matrix $M(g)$ of $\overline{G}$ corresponding to the inertia factor H_5 has the form $\left[a^{1_1}_{(5,y_1)}, -a^{1_1}_{(5,y_1)}, 0, ..., 0\right]$. The entry $a^{1_1}_{(5,y_1)} = \sqrt{\frac{|C_{\overline{G_1}}(\overline{q_1})|}{|C_G(g)|}} = 2^m$ for $m \in \{1, 2, \ldots 16\}$.

Example: Lifting of a Fischer matrix of $\overline{G_1}$ to a Fischer matrix of $\overline{G}$

	$\lvert C_{\overline{G_1}}(\overline{q_1})\rvert$	$\lvert C_{\overline{F}}(\overline{q_2})\rvert$	$\lvert C_{\overline{F}}(\overline{q_3})\rvert$	$\lvert C_{\overline{F}}(\overline{q_4})\rvert$
$\lvert C_{H_1}(y_1 \in 1A)\rvert$	1	1	1	1
$\lvert C_{H_2}(y_1 \in 1A)\rvert$	32640	−128	−128	128
$\lvert C_{H_3}(y_2 \in 1A)\rvert$	30600	−120	136	−120
$\lvert C_{H_4}(y_1 \in 1A)\rvert$	2295	247	−9	−9

Figure: $\widehat{M(1A)}$ of $\overline{G_1}$

		$X(\overline{q_1})$	$X(\overline{q_2})$	$X(\overline{q_3})$	$X(\overline{q_4})$
	$\lvert C_{\overline{G}}(\overline{g_{11}})\rvert$	$\lvert C_{\overline{G}}(\overline{g_{12}})\rvert$	$\lvert C_{\overline{G}}(\overline{g_{21}})\rvert$	$\lvert C_{\overline{G}}(\overline{g_{31}})\rvert$	$\lvert C_{\overline{G}}(\overline{g_{41}})\rvert$
$\lvert C_{H_1}(y_1 \in 1A)\rvert$	1	1	1	1	1
$\lvert C_{H_2}(y_1 \in 1A)\rvert$	32640	32640	−128	−128	128
$\lvert C_{H_3}(y_2 \in 1A)\rvert$	30600	30600	−120	136	−120
$\lvert C_{H_4}(y_1 \in 1A)\rvert$	2295	2295	247	−9	−9
$\lvert C_{H_5}(y_1 \in 1A)\rvert$	256	−256	0	0	0

Figure: $M(1A)$ of $\overline{G}$

Fischer-Clifford Matrices of and $\overline{G_1}$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 32640 & -128 & -128 & 128 \\ 30600 & -120 & 136 & -120 \\ 2295 & 247 & -9 & -9 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1920 & -128 & -128 & 128 & 0 & 0 \\ 360 & 104 & -24 & -24 & -24 & 8 \\ 1440 & -96 & 160 & -96 & 0 & 0 \\ 15 & 15 & 15 & 15 & -1 & -1 \\ 360 & 104 & -24 & -24 & 24 & -8 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 \\ 120 & 8 & -8 \\ 135 & -9 & 7 \end{pmatrix}$	$M(2C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 32 & -32 & -32 & 32 & 0 & 0 & 0 \\ 480 & 160 & -96 & -32 & 0 & 0 & 0 \\ 360 & -120 & 72 & -24 & -24 & 8 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 \\ 120 & -40 & 24 & -8 & 24 & -8 & 0 \\ 30 & 30 & 30 & 30 & -2 & -2 & 0 \end{pmatrix}$
$M(2D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 120 & -8 & -8 & 8 \\ 15 & 15 & -1 & -1 \\ 120 & -8 & 8 & -8 \end{pmatrix}$	$M(2E) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 128 & -128 & 0 & 0 & 0 & 0 & 0 & 0 \\ 24 & 24 & 24 & -8 & -8 & 8 & 0 & 0 \\ 64 & 64 & -64 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\ 24 & 24 & 24 & -8 & 8 & -8 & 0 & 0 \\ 12 & 12 & 12 & 12 & -4 & -4 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 & -2 & 0 \end{pmatrix}$

Fischer-Clifford Matrices of $\overline{G_1}$

$M(g)$	$M(g)$
$M(2F) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 24 & 24 & 8 & -8 & 0 \\ 192 & -64 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & -1 \\ 36 & 36 & -12 & 4 & 0 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{pmatrix}$
$M(3B) = \begin{pmatrix} 1 & 1 & 1 \\ 36 & 4 & -4 \\ 27 & -5 & 3 \end{pmatrix}$	$M(3C) = \begin{pmatrix} 1 \end{pmatrix}$
$M(3D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 120 & -8 & -8 & 8 \\ 90 & -6 & 10 & -6 \\ 45 & 13 & -3 & -3 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & -4 & 4 & 4 & -4 & 0 \\ 24 & -24 & 0 & -8 & 8 & 0 & 0 \\ 8 & -8 & 0 & 8 & -8 & 0 & 0 \\ 12 & 12 & 12 & -4 & -4 & -4 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 & -1 \\ 12 & 12 & -12 & -4 & -4 & 4 & 0 \end{pmatrix}$
$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 96 & -32 & 32 & -32 & 0 & 0 \\ 48 & 48 & -16 & -16 & 0 & 0 \\ 96 & -32 & -32 & 32 & 0 & 0 \\ 6 & 6 & 6 & 6 & 2 & -2 \\ 9 & 9 & 9 & 9 & -3 & 1 \end{pmatrix}$	$M(4C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & -4 & 4 & -4 & -4 & 4 & 0 & 0 & 0 \\ 24 & 24 & -8 & -8 & 0 & 0 & 0 & 0 & 0 \\ 4 & -4 & 4 & -4 & 4 & -4 & 0 & 0 & 0 \\ 12 & -12 & -4 & 4 & 0 & 0 & -4 & 4 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 12 & -12 & -4 & 4 & 0 & 0 & 4 & -4 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 & -1 & -1 & -1 \\ 3 & 3 & 3 & 3 & -3 & -3 & -1 & -1 & 1 \end{pmatrix}$

Fischer-Clifford Matrices of and $\overline{G_1}$

$M(g)$	$M(g)$
$M(4D) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$	$M(4E) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$
$M(4F) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 & 0 \\ 4 & 4 & -4 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 2 & 2 & 2 & -2 & 0 \end{pmatrix}$	$M(4G) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 \\ 6 & 6 & -2 & 0 \\ 1 & 1 & 1 & -1 \end{pmatrix}$
$M(4H) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 & 0 \\ 2 & 2 & 2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 4 & 4 & -4 & 0 & 0 \end{pmatrix}$	$M(4I) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 \\ 6 & 6 & -2 & 0 \\ 1 & 1 & 1 & -1 \end{pmatrix}$
$M(4J) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 96 & -32 & -32 & 32 & 0 & 0 & 0 & 0 \\ 48 & 48 & -16 & -16 & 0 & 0 & 0 & 0 \\ 48 & -16 & 16 & -16 & -8 & 8 & 0 & 0 \\ 6 & 6 & 6 & 6 & -2 & -2 & -2 & 2 \\ 3 & 3 & 3 & 3 & -1 & -1 & 3 & -1 \\ 6 & 6 & 6 & 6 & 2 & 2 & -2 & -2 \\ 48 & -16 & 16 & -16 & 8 & -8 & 0 & 0 \end{pmatrix}$	$M(4K) = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 12 & -4 & 0 \end{pmatrix}$
$M(4L) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & -4 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 8 & -8 & 0 & 0 & 0 \\ 2 & 2 & 2 & -2 & 0 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 & 1 \\ 10 & -2 & 2 \\ 5 & 1 & -3 \end{pmatrix}$

Fischer-Clifford Matrices of $\overline{G_1}$

$M(g)$	$M(g)$	$M(g)$
$M(5B) = (1)$	$M(6A) = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{pmatrix}$	$M(6B) = (1)$
$M(6C) = \begin{pmatrix} 1 & 1 & 1 \\ 12 & -4 & 0 \\ 3 & 3 & -1 \end{pmatrix}$	$M(6D) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & -2 & 2 & -2 & 0 \\ 6 & 6 & -2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 6 & -6 & -2 & 2 & 0 \end{pmatrix}$	$M(6E) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 24 & 8 & -8 & 0 & 0 \\ 18 & -6 & 2 & -6 & 2 \\ 3 & 3 & 3 & -1 & -1 \\ 18 & -6 & 2 & 6 & -2 \end{pmatrix}$
$M(6F) = (1)$	$M(6G) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 6 & -2 & 2 & -2 \\ 3 & 3 & -1 & -1 \\ 6 & -2 & -2 & 2 \end{pmatrix}$	$M(6H) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 & 0 \\ 4 & 4 & -4 & 0 & 0 \\ 2 & 2 & 2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \end{pmatrix}$
$M(6I) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(6J) = (1)$	$M(6K) = (1)$
$M(6L) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(6M) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$	$M(6N) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 \\ 6 & 6 & -2 & 0 \\ 1 & 1 & 1 & -1 \end{pmatrix}$

Fischer-Clifford Matrices of $\overline{G_1}$

$M(g)$	$M(g)$	$M(g)$
$M(6O) = \begin{pmatrix} 1 & 1 & 1 \\ 12 & -4 & 0 \\ 3 & 3 & -1 \end{pmatrix}$	$M(6P) = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{pmatrix}$	$M(7A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -2 & -2 & -2 & 2 \\ 3 & -1 & -1 & 3 & -1 \\ 3 & 3 & -1 & -1 & -1 \\ 3 & -1 & 3 & -1 & -1 \end{pmatrix}$
$M(8A) = \begin{pmatrix} 1 & 1 & 1 \\ 12 & -4 & 0 \\ 3 & 3 & -1 \end{pmatrix}$	$M(8B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 & 0 \\ 4 & 4 & -4 & 0 & 0 \\ 2 & 2 & 2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \end{pmatrix}$	$M(8C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(8D) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 & 0 \\ 4 & 4 & -4 & 0 & 0 \\ 2 & 2 & 2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \end{pmatrix}$	$M(8E) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(8F) = \begin{pmatrix} 1 & 1 & 1 \\ 12 & -4 & 0 \\ 3 & 3 & -1 \end{pmatrix}$

Fischer-Clifford Matrices of $\overline{G_1}$

$M(g)$	$M(g)$	$M(g)$
$M(9A) = (1)$	$M(9B) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(10A) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$
$M(10B) = (1)$	$M(10C) = (1)$	$M(10D) = (1)$
$M(12A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(12B) = (1)$	$M(12C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -2 & -2 & -2 & 2 \\ 3 & -1 & 3 & -1 & -1 \\ 3 & 3 & -1 & -1 & -1 \\ 3 & -1 & -1 & 3 & -1 \end{pmatrix}$
$M(12D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(12E) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(12F) = (1)$
$M(12G) = (1)$	$M(12H) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$	$M(12I) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$

Fischer-Clifford Matrices of $\overline{G_1}$

$M(g)$	$M(g)$	$M(g)$
$M(12J) = \begin{pmatrix}1 & 1 & 1\\ 2 & -2 & 0\\ 1 & 1 & -1\end{pmatrix}$	$M(12K) = (1)$	$M(12L) = \begin{pmatrix}1 & 1\\ 3 & -1\end{pmatrix}$
$M(12M) = (1)$	$M(14A) = \begin{pmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\\ 1 & -1 & 1 & -1\end{pmatrix}$	$M(15A) = (1)$
$M(15B) = (1)$	$M(15C) = \begin{pmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\\ 1 & -1 & 1 & -1\end{pmatrix}$	$M(17A) = (1)$
$M(17B) = (1)$	$M(18A) = (1)$	$M(20A) = (1)$
$M(20B) = (1)$	$M(21A) = (1)$	$M(24A) = (1)$
$M(24B) = (1)$	$M(30A) = (1)$	$M(30B) = (1)$

Partial Character Table

The partial character table of $\overline{G}$ on the classes from the coset $P_1\overline{g}$ with class representatives in the set $\overline{X(g)}$, is given by

$$\begin{bmatrix} C_1(g) & M_1(g) \\ C_2(g) & M_2(g) \\ C_3(g) & M_3(g) \\ C_4(g) & M_4(g) \\ C_5(g) & M_5(g) \end{bmatrix}.$$

Figure: Partial Character Table corresponding to $P_1\overline{g}$

Full ordinary character table of $\overline{G}$

The full ordinary character table of $\overline{G}$ will have the structure as represented below.

$$\left[\begin{array}{c|c|c|c} C_1(1)M_1(1) & C_1(g_2)M_1(g_2) & \ldots & C_1(g_{81})M_1(g_{81}) \\ \hline C_2(1)M_2(1) & C_2(g_2)M_2(g_2) & \ldots & C_2(g_{81})M_2(g_{81}) \\ \hline C_3(1)M_3(1) & C_3(g_2)M_3(g_2) & \ldots & C_3(g_{81})M_3(g_{81}) \\ \hline C_4(1)M_4(1) & C_4(g_2)M_4(g_2) & \ldots & C_4(g_{81})M_4(g_{81}) \\ C_5(1)M_5(1) & C_5(g_2)M_5(g_2) & \ldots & C_5(g_{81})M_5(g_{81}) \\ \hline \end{array}\right]$$

Figure: Structure of the Character Table of $\overline{G}$

The character table of $\overline{G}$ will be a 326×326 matrix with complex entries.

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