On Vanishing Conjugacy Classes in Finite Groups

Roghayeh HAFEZIEH

Gebze Technical University, Gebze, Turkey

Outline

Outline

- Preleminaries

Outline

- Preleminaries
- Groups whose vanishing class sizes are prime powers

Outline

- Preleminaries
- Groups whose vanishing class sizes are prime powers
- Groups whose vanishing class sizes are p-powers

Outline

- Preleminaries
- Groups whose vanishing class sizes are prime powers
- Groups whose vanishing class sizes are p-powers

Definition

Definition

In this talk, all groups are considered to be finite.

Definition

In this talk, all groups are considered to be finite.
Given a finite group G,

Definition

In this talk, all groups are considered to be finite.
Given a finite group G,

- an element is called a vanishing element of G if it is a zero of an irreducible character of G;

Definition

In this talk, all groups are considered to be finite.
Given a finite group G,

- an element is called a vanishing element of G if it is a zero of an irreducible character of G;
- otherwise, it is called a non-vanishing element.

Definition

In this talk, all groups are considered to be finite.

Given a finite group G,

- an element is called a vanishing element of G if it is a zero of an irreducible character of G;
- otherwise, it is called a non-vanishing element.
- the conjugacy class of an element is termed as a vanishing class if that element is a vanishing element.

Definition

In this talk, all groups are considered to be finite.
Given a finite group G,

- an element is called a vanishing element of G if it is a zero of an irreducible character of G;
- otherwise, it is called a non-vanishing element.
- the conjugacy class of an element is termed as a vanishing class if that element is a vanishing element.

Influence of the conjugacy class sizes and vanishing class sizes

Influence of the conjugacy class sizes and vanishing class sizes

- A prime $p \in \pi(G)$ does not divide any conjugacy class size of G if and only if G has a central Sylow p-subgroup.

Influence of the conjugacy class sizes and vanishing class sizes

- A prime $p \in \pi(G)$ does not divide any conjugacy class size of G if and only if G has a central Sylow p-subgroup.
- **Dolfi, Pacifici, and Sanus in 2010**, considering the case where no vanishing class of G is divisible by a given prime p, proved that G has a normal p-complement and abelian Sylow p-subgroups.

Influence of the conjugacy class sizes and vanishing class sizes

- A prime $p \in \pi(G)$ does not divide any conjugacy class size of G if and only if G has a central Sylow p-subgroup.
- **Dolfi, Pacifici, and Sanus in 2010**, considering the case where no vanishing class of G is divisible by a given prime p, proved that G has a normal p-complement and abelian Sylow p-subgroups.
- **Chillag and Herzog in 1990** proved that if the size of each conjugacy class of G is square-free, then G is supersolvable.

Influence of the conjugacy class sizes and vanishing class sizes

- A prime $p \in \pi(G)$ does not divide any conjugacy class size of G if and only if G has a central Sylow p-subgroup.
- **Dolfi, Pacifici, and Sanus in 2010**, considering the case where no vanishing class of G is divisible by a given prime p, proved that G has a normal p-complement and abelian Sylow p-subgroups.
- **Chillag and Herzog in 1990** proved that if the size of each conjugacy class of G is square-free, then G is supersolvable.
- **Brough in 2016** proved that a group whose vanishing class sizes are all square-free numbers is supersolvable.

Influence of the conjugacy class sizes and vanishing class sizes

- A prime $p \in \pi(G)$ does not divide any conjugacy class size of G if and only if G has a central Sylow p-subgroup.
- **Dolfi, Pacifici, and Sanus in 2010**, considering the case where no vanishing class of G is divisible by a given prime p, proved that G has a normal p-complement and abelian Sylow p-subgroups.
- **Chillag and Herzog in 1990** proved that if the size of each conjugacy class of G is square-free, then G is supersolvable.
- **Brough in 2016** proved that a group whose vanishing class sizes are all square-free numbers is supersolvable.
- **Chillag and Herzog, in 1990** showed that if the size of each conjugacy class of G is a prime power, then G is either nilpotent or an A-group which is a solvable group whose Sylow subgroups are all abelian.

Influence of the conjugacy class sizes and vanishing class sizes

- A prime $p \in \pi(G)$ does not divide any conjugacy class size of G if and only if G has a central Sylow p-subgroup.
- **Dolfi, Pacifici, and Sanus in 2010**, considering the case where no vanishing class of G is divisible by a given prime p, proved that G has a normal p-complement and abelian Sylow p-subgroups.
- **Chillag and Herzog in 1990** proved that if the size of each conjugacy class of G is square-free, then G is supersolvable.
- **Brough in 2016** proved that a group whose vanishing class sizes are all square-free numbers is supersolvable.
- **Chillag and Herzog, in 1990** showed that if the size of each conjugacy class of G is a prime power, then G is either nilpotent or an A-group which is a solvable group whose Sylow subgroups are all abelian.

Solvability Criterion

Solvability Criterion

Robati-Hafezieh, 2022

Proposition

Let S be a sporadic simple group or an alternating group Alt(n) for some $n \geqslant 7$. Then S has an irreducible character θ which extends to Aut(S) and a conjugacy class x^S of size divisible by $2q$ for some odd prime number q such that $\theta(x) = 0$.

Solvability Criterion

Robati-Hafezieh, 2022

Proposition

Let S be a sporadic simple group or an alternating group Alt(n) for some $n \geqslant 7$. Then S has an irreducible character θ which extends to Aut(S) and a conjugacy class x^S of size divisible by $2q$ for some odd prime number q such that $\theta(x) = 0$.

Bianchi-Chillag-Lewis-Pacifici, 2007

Lemma

Let G be a group, and $M = S_1 \times \ldots \times S_k$ be a minimal normal subgroup of G, where every S_i is isomorphic to a non-abelian simple group S. If $\theta \in \mathrm{Irr}(S)$ extends to $\mathrm{Aut}(S)$, then $\theta \times \ldots \times \theta \in \mathrm{Irr}(M)$ extends to G.

Solvability Criterion

Robati-Hafezieh, 2022

Proposition

Let S be a sporadic simple group or an alternating group Alt(n) for some $n \geqslant 7$. Then S has an irreducible character θ which extends to Aut(S) and a conjugacy class x^S of size divisible by $2q$ for some odd prime number q such that $\theta(x) = 0$.

Bianchi-Chillag-Lewis-Pacifici, 2007

Lemma

Let G be a group, and $M = S_1 \times \ldots \times S_k$ be a minimal normal subgroup of G, where every S_i is isomorphic to a non-abelian simple group S. If $\theta \in \mathrm{Irr}(S)$ extends to $\mathrm{Aut}(S)$, then $\theta \times \ldots \times \theta \in \mathrm{Irr}(M)$ extends to G.

Solvability Criterion

Solvability Criterion

Robati-Hafezieh, 2022

Theorem

Let G be a finite group. If all vanishing class sizes of G are either odd or powers of 2, then G is solvable.

Solvability Criterion

Robati-Hafezieh, 2022

Theorem

Let G be a finite group. If all vanishing class sizes of G are either odd or powers of 2, then G is solvable.

Groups whose vanishing classes all have size p

Groups whose vanishing classes all have size p

Bianchi-Lewis-Pacifici, 2020

Theorem

Let G be a group and p be a prime. The vanishing class sizes of G all have size p if and only if one of the following occurs:

Bianchi-Lewis-Pacifici, 2020

Theorem

Let G be a group and p be a prime. The vanishing class sizes of G all have size p if and only if one of the following occurs:

- $G = P \times H$, *where* P *is a* p*-group whose conjugacy class sizes are* 1 *or* p, *and* H *is an abelian* p'*-group.*

Groups whose vanishing classes all have size p

Bianchi-Lewis-Pacifici, 2020

Theorem

Let G be a group and p be a prime. The vanishing class sizes of G all have size p if and only if one of the following occurs:

- $G = P \times H$, *where* P *is a* p*-group whose conjugacy class sizes are* 1 *or* p, *and* H *is an abelian* p'*-group.*
- $\frac{G}{Z(G)}$ *is a Frobenius group whose Frobenius kernel has order* p.

Groups whose vanishing classes all have size p

Bianchi-Lewis-Pacifici, 2020

Theorem

Let G be a group and p be a prime. The vanishing class sizes of G all have size p if and only if one of the following occurs:

- $G = P \times H$, *where* P *is a* p*-group whose conjugacy class sizes are* 1 *or* p, *and* H *is an abelian* p'*-group.*
- $\frac{G}{Z(G)}$ *is a Frobenius group whose Frobenius kernel has order* p.

Groups with a unique vanishing class size

Bianchi-Camina-Lewis-Pacifici, 2020

Theorem

Let G *be a finite supersolvable group, and assume that* $\mathrm{Vcs}(G) = \{s\}$*. If, denoting by* π *the set of prime divisors of* s*, we have* $\pi = \pi(G/Z(G))$*, then, up to an abelian direct factor of* π'*-order, one of the following occurs:*

Groups with a unique vanishing class size

Bianchi-Camina-Lewis-Pacifici, 2020

Theorem

Let G *be a finite supersolvable group, and assume that* $\mathrm{Vcs}(G) = \{s\}$*. If, denoting by* π *the set of prime divisors of* s*, we have* $\pi = \pi(G/Z(G))$*, then, up to an abelian direct factor of* π'*-order, one of the following occurs:*

- *The integer* s *is power of a suitable prime* p*,* G *is a* p*-group and* $\mathrm{cs}(G) = \{1, s\}$*.*

Groups with a unique vanishing class size

Bianchi-Camina-Lewis-Pacifici, 2020

Theorem

Let G *be a finite supersolvable group, and assume that* $\mathrm{Vcs}(G) = \{s\}$*. If, denoting by* π *the set of prime divisors of* s*, we have* $\pi = \pi(G/Z(G))$*, then, up to an abelian direct factor of* π'*-order, one of the following occurs:*

- *The integer* s *is power of a suitable prime* p*,* G *is a* p*-group and* $\mathrm{cs}(G) = \{1, s\}$*.*
- *For a suitable prime* p *we have* $G = NP$*, where* N *is a nontrivial, abelian, normal* p*- complement of* G*, and* P *is a Sylow* p*-subgroup of* G *such that* $|\mathrm{cs}(P)| = 2$*,* $Z(P) = C_P(N)$*, and* $P/Z(P)$ *is an elementary abelian* p*-group. Also,* $|N : C_N(x)|$ *has the same value for every element* $x \in P \setminus Z(P)$*,* $C_N(P) = N \cap Z(G)$*, and* N *contains no vanishing elements of* G*.*

Groups with a unique vanishing class size

Bianchi-Camina-Lewis-Pacifici, 2020

Theorem

Let G be a finite supersolvable group, and assume that $\mathrm{Vcs}(G) = \{s\}$. If, denoting by π the set of prime divisors of s, we have $\pi = \pi(G/Z(G))$, then, up to an abelian direct factor of π'-order, one of the following occurs:

- *The integer s is power of a suitable prime p, G is a p-group and $\mathrm{cs}(G) = \{1, s\}$.*
- *For a suitable prime p we have $G = NP$, where N is a nontrivial, abelian, normal p- complement of G, and P is a Sylow p-subgroup of G such that $|\mathrm{cs}(P)| = 2$, $Z(P) = C_P(N)$, and $P/Z(P)$ is an elementary abelian p-group. Also, $|N : C_N(x)|$ has the same value for every element $x \in P \setminus Z(P)$, $C_N(P) = N \cap Z(G)$, and N contains no vanishing elements of G.*

Characterization Theorem

Characterization Theorem

Robati-Hafezieh,2022

Theorem

Let G be a finite group. If each vanishing class size of G is a prime power, then one of the following situations occurs:

Characterization Theorem

Robati-Hafezieh,2022

Theorem

Let G be a finite group. If each vanishing class size of G is a prime power, then one of the following situations occurs:

(1) $G/F(G)$ *is a nilpotent group with at most one non-abelian Sylow subgroup and* $|\rho(cs(G/F(G)))| \leqslant 1$.

Characterization Theorem

Robati-Hafezieh,2022

Theorem

Let G be a finite group. If each vanishing class size of G is a prime power, then one of the following situations occurs:

(1) $G/F(G)$ *is a nilpotent group with at most one non-abelian Sylow subgroup and* $|\rho(cs(G/F(G)))| \leqslant 1$.

(2) $G/F(G)$ *is an* A*-group and there exist two distinct primes* r *and* s *such that* $G/F(G) \cong K \times H$, *where* K *is an abelian* $\{r, s\}'$*-group and* $H/Z(H)$ *is a Frobenius* $\{r, s\}$*-group. Moreover,* $|\rho(cs(G/F(G)))| = dl(G/F(G)) = 2$.

Characterization Theorem

Robati-Hafezieh,2022

Theorem

Let G be a finite group. If each vanishing class size of G is a prime power, then one of the following situations occurs:

(1) $G/F(G)$ *is a nilpotent group with at most one non-abelian Sylow subgroup and* $|\rho(cs(G/F(G)))| \leqslant 1$.

(2) $G/F(G)$ *is an* A*-group and there exist two distinct primes* r *and* s *such that* $G/F(G) \cong K \times H$, *where* K *is an abelian* $\{r, s\}'$*-group and* $H/Z(H)$ *is a Frobenius* $\{r, s\}$*-group. Moreover,* $|\rho(cs(G/F(G)))| = dl(G/F(G)) = 2$.

Robati-Hafezieh,2023

Theorem

Let G *be a finite group. If the image of each vanishing element of* G *in* $G/F(G)$ *has a prime power index, then one of the following situations occurs:*

Generalized version of the previous Theorem

Robati-Hafezieh,2023

Theorem

Let G be a finite group. If the image of each vanishing element of G in $G/F(G)$ has a prime power index, then one of the following situations occurs:

(1) $G/F(G)$ *is a nilpotent group with at most one non-abelian Sylow subgroup and* $|\rho(cs(G/F(G)))| \leqslant 1$.

Generalized version of the previous Theorem

Robati-Hafezieh,2023

Theorem

Let G be a finite group. If the image of each vanishing element of G in $G/F(G)$ has a prime power index, then one of the following situations occurs:

(1) $G/F(G)$ *is a nilpotent group with at most one non-abelian Sylow subgroup and* $|\rho(cs(G/F(G)))| \leqslant 1$.

(2) $G/F(G)$ *is an* A*-group and there exist two distinct primes* r *and* s *such that* $G/F(G) \cong K \times H$, *where* K *is an abelian* $\{r, s\}'$*-group and* $H/Z(H)$ *is a Frobenius* $\{r, s\}$*-group. Moreover,* $|\rho(cs(G/F(G)))| = dl(G/F(G)) = 2$.

Generalized version of the previous Theorem

Robati-Hafezieh,2023

Theorem

Let G be a finite group. If the image of each vanishing element of G in $G/F(G)$ has a prime power index, then one of the following situations occurs:

(1) $G/F(G)$ *is a nilpotent group with at most one non-abelian Sylow subgroup and* $|\rho(cs(G/F(G)))| \leqslant 1$.

(2) $G/F(G)$ *is an* A*-group and there exist two distinct primes* r *and* s *such that* $G/F(G) \cong K \times H$, *where* K *is an abelian* $\{r, s\}'$*-group and* $H/Z(H)$ *is a Frobenius* $\{r, s\}$*-group. Moreover,* $|\rho(cs(G/F(G)))| = dl(G/F(G)) = 2$.

The second case of the main theorem mentioned by Robati-Hafezien in 2022 does not occur

The second case of the main theorem mentioned by Robati-Hafezien in 2022 does not occur

Robati-Hafezieh,2023:

Corollary

Let G be a finite group whose vanishing class sizes are all prime powers. Then $G/F(G)$ is abelian.

The second case of the main theorem mentioned by Robati-Hafezien in 2022 does not occur

Robati-Hafezieh,2023:

Corollary

Let G be a finite group whose vanishing class sizes are all prime powers. Then $G/F(G)$ is abelian.

An answer to the conjecture mentioned by Isaacs, Navarro and Wolf

An answer to the conjecture mentioned by Isaacs, Navarro and Wolf

Isaacs-Navarro-Wolf, 1999

Conjecture

Each non-vanishing element of a finite solvable group G lies in the Fitting subgroup $F(\mathrm{G})$.

An answer to the conjecture mentioned by Isaacs, Navarro and Wolf

Isaacs-Navarro-Wolf, 1999

Conjecture

Each non-vanishing element of a finite solvable group G lies in the Fitting subgroup $F(G)$.

Robati-Hafezieh,2022

Corollary

Let G be a finite group whose vanishing conjugacy classes sizes are of prime powers. If x is a non-vanishing element, then $x \in F(G)$.

An answer to the conjecture mentioned by Isaacs, Navarro and Wolf

Isaacs-Navarro-Wolf, 1999

Conjecture

Each non-vanishing element of a finite solvable group G lies in the Fitting subgroup $F(G)$.

Robati-Hafezieh,2022

Corollary

Let G be a finite group whose vanishing conjugacy classes sizes are of prime powers. If x is a non-vanishing element, then $x \in F(G)$.

Ahanjideh-Robati, 2023

Theorem

For every finite group G with exactly one vanishing class size,
$G \setminus F(G) \subseteq Van(G)$.

An answer to the conjecture mentioned by Isaacs, Navarro and Wolf

Isaacs-Navarro-Wolf, 1999

Conjecture

Each non-vanishing element of a finite solvable group G lies in the Fitting subgroup $F(G)$.

Robati-Hafezieh,2022

Corollary

Let G be a finite group whose vanishing conjugacy classes sizes are of prime powers. If x is a non-vanishing element, then $x \in F(G)$.

Ahanjideh-Robati, 2023

Theorem

For every finite group G with exactly one vanishing class size,
$G \setminus F(G) \subseteq Van(G)$.

The existence of a normal Sylow subgroup

Dolfi-Pacifici-Sanus, 2010

Theorem

Let G be a finite group, and q a prime number. If the size of every vanishing conjugacy class of G is not divisible by q, then G has a normal q-complement and abelian Sylow q-subgroups.

The existence of a normal Sylow subgroup

Dolfi-Pacifici-Sanus, 2010

Theorem

Let G be a finite group, and q a prime number. If the size of every vanishing conjugacy class of G is not divisible by q, then G has a normal q-complement and abelian Sylow q-subgroups.

The above theorem implies that:

The existence of a normal Sylow subgroup

Dolfi-Pacifici-Sanus, 2010

Theorem

Let G be a finite group, and q a prime number. If the size of every vanishing conjugacy class of G is not divisible by q, then G has a normal q-complement and abelian Sylow q-subgroups.

The above theorem implies that: If G is a finite group whose vanishing class sizes are all powers of a prime p, then $G = PH$, where P is the normal Sylow p-subgroup and H is an abelian p'-subgroup of G.

The existence of a normal Sylow subgroup

Dolfi-Pacifici-Sanus, 2010

Theorem

Let G be a finite group, and q a prime number. If the size of every vanishing conjugacy class of G is not divisible by q, then G has a normal q-complement and abelian Sylow q-subgroups.

The above theorem implies that: If G is a finite group whose vanishing class sizes are all powers of a prime p, then $G = PH$, where P is the normal Sylow p-subgroup and H is an abelian p'-subgroup of G.

Abelian or non-abelian normal Sylow p-subgroup

Abelian or non-abelian normal Sylow p-subgroup

Robati-Hafezieh,2023

Lemma

Let G be a finite group whose vanishing class sizes are all powers of a prime p. Let P be the normal Sylow p-subgroup of G. If P is abelian, then G is either abelian or a quasi-Frobenius group.

Abelian or non-abelian normal Sylow p-subgroup

Robati-Hafezieh,2023

Lemma

Let G be a finite group whose vanishing class sizes are all powers of a prime p. Let P be the normal Sylow p-subgroup of G. If P is abelian, then G is either abelian or a quasi-Frobenius group.

Abelian or non-abelian normal Sylow p-subgroup

Robati-Hafezieh,2023

Lemma

Let G be a finite group whose vanishing class sizes are all powers of a prime p. Let P be the normal Sylow p-subgroup of G. If P is abelian, then G is either abelian or a quasi-Frobenius group.

Lemma

Let G be a finite group whose vanishing class sizes are all powers of a prime p. Let P be the normal Sylow p-subgroup of G. If P is non-abelian, then for each $x \in P$ either $C_G(x) \subseteq F(G)$ or $x \subseteq C_P(H)^g$ for a suitable $g \in P$.

Abelian or non-abelian normal Sylow p-subgroup

Robati-Hafezieh,2023

Lemma

Let G be a finite group whose vanishing class sizes are all powers of a prime p. Let P be the normal Sylow p-subgroup of G. If P is abelian, then G is either abelian or a quasi-Frobenius group.

Lemma

Let G be a finite group whose vanishing class sizes are all powers of a prime p. Let P be the normal Sylow p-subgroup of G. If P is non-abelian, then for each $x \in P$ either $C_G(x) \subseteq F(G)$ or $x \subseteq C_P(H)^g$ for a suitable $g \in P$.

Robati-Hafezieh,2023

Theorem

Let G be a finite group whose vanishing class sizes are p-powers, for a prime p.

Robati-Hafezieh,2023

Theorem

Let G be a finite group whose vanishing class sizes are p-powers, for a prime p. Then $G = PH$, where P is a normal subgroup and H is an abelian p'-subgroup of G.

p-power vanishing class sizes

Robati-Hafezieh,2023

Theorem

Let G be a finite group whose vanishing class sizes are p-powers, for a prime p. Then $G = PH$, where P is a normal subgroup and H is an abelian p'-subgroup of G.
Indeed, if G is non-abelian, then either P is abelian and G is a quasi-Frobenius group, or P is non-abelian and one of the following situation occurs:

p-power vanishing class sizes

Robati-Hafezieh,2023

Theorem

Let G *be a finite group whose vanishing class sizes are* p*-powers, for a prime* p*. Then* $G = PH$*, where* P *is a normal subgroup and* H *is an abelian* p'*-subgroup of* G*.*
Indeed, if G *is non-abelian, then either* P *is abelian and* G *is a quasi-Frobenius group, or* P *is non-abelian and one of the following situation occurs:*

p-power vanishing class sizes

p-power vanishing class sizes

(1) $G = P \times H$.

p-power vanishing class sizes

(1) $G = P \times H$.

(2) $G = O^p(G) \rtimes C_P(H)$, where $O^p(G) = G' \rtimes H$, $G' = [P, H]$, and the following properties hold:

p-power vanishing class sizes

(1) $G = P \times H$.

(2) $G = O^p(G) \rtimes C_P(H)$, where $O^p(G) = G' \rtimes H$, $G' = [P, H]$, and the following properties hold:

 (a) $C_H(P) = F(G)_{p'} = Z(G)_{p'}$;
 (b) $C_P(H)$ is abelian;
 (c) $O^p(G)$ is a quasi-Frobenius group;
 (d) $P \setminus \bigcup_{g \in P} C_P(H)^g \cap \mathrm{Van}(G) = \emptyset$. In particular, all elements of G' are non-vanishing elements of G;
 (e) $G/F(G)$ is cyclic.

p-power vanishing class sizes

(1) $G = P \times H$.

(2) $G = O^p(G) \rtimes C_P(H)$, where $O^p(G) = G' \rtimes H$, $G' = [P, H]$, and the following properties hold:

  (a) $C_H(P) = F(G)_{p'} = Z(G)_{p'}$;
  (b) $C_P(H)$ is abelian;
  (c) $O^p(G)$ is a quasi-Frobenius group;
  (d) $P \setminus \bigcup_{g \in P} C_P(H)^g \cap \mathrm{Van}(G) = \emptyset$. In particular, all elements of G' are non-vanishing elements of G;
  (e) $G/F(G)$ is cyclic.

Vanishing class sizes of G and its associated prime graph

Vanishing class sizes of G and its associated prime graph

As a corollary of the main theorem mentioned by **Dolfi, Pacifici, Sanus, in 2010** we have:

Corollary

Given a group G*, if a prime* p *is not a vertex of the prime graph associated to* $Vcs(G)$*, then* G *has a normal* p*-complement and abelian Sylow* p*-subgroups.*

Vanishing class sizes of G and its associated prime graph

As a corollary of the main theorem mentioned by **Dolfi, Pacifici, Sanus, in 2010** we have:

Corollary

Given a group G, if a prime p is not a vertex of the prime graph associated to $Vcs(G)$, then G has a normal p-complement and abelian Sylow p-subgroups.

Bianchi, Brough, Camina, and Pacifici, in 2017

Theorem

Given a finite group G with $F(G) = 1$, the prime graph associated to $Vcs(G)$ is a complete graph with vertex set $\pi(G)$.

Vanishing class sizes of G and its associated prime graph

As a corollary of the main theorem mentioned by **Dolfi, Pacifici, Sanus, in 2010** we have:

Corollary

Given a group G*, if a prime* p *is not a vertex of the prime graph associated to* $Vcs(G)$*, then* G *has a normal* p*-complement and abelian Sylow* p*-subgroups.*

Bianchi, Brough, Camina, and Pacifici, in 2017

Theorem

Given a finite group G *with* $F(G) = 1$*, the prime graph associated to* $Vcs(G)$ *is a complete graph with vertex set* $\pi(G)$*.*

Hence the Theorem mentioned by **Robati-Hafezieh, 2023** give a classification for those finite groups whose prime graph associated to the set of vanishing class sizes, is a **singleton**.

Vanishing class sizes of G and its associated prime graph

As a corollary of the main theorem mentioned by **Dolfi, Pacifici, Sanus, in 2010** we have:

Corollary

Given a group G, if a prime p is not a vertex of the prime graph associated to $Vcs(G)$, then G has a normal p-complement and abelian Sylow p-subgroups.

Bianchi, Brough, Camina, and Pacifici, in 2017

Theorem

Given a finite group G with $F(G) = 1$, the prime graph associated to $Vcs(G)$ is a complete graph with vertex set $\pi(G)$.

Hence the Theorem mentioned by **Robati-Hafezieh, 2023** give a classification for those finite groups whose prime graph associated to the set of vanishing class sizes, is a **singleton**.

bibliography

bibliography

- N. Ahanjideh, S. Mahmood Robati, *The Isaacs-Navarro-Wolf conjecture for groups with one vanishing class size*, *Mediterr. J. Math.* , (2023).
- M. Bianchi, R. D. Camina, M. L. Lewis, and E. Pacifici, *On solvable groups with one vanishing class size*, *Proceedings of the Royal Society of Edinburgh Section A*: Mathematics,(2020), 1-20.
- M. Bianchi, M. L. Lewis, and E. Pacifici,*Groups with vanishing class size* p, *J. Group Theory*, **23**, (2020), 79-83.
- M. Bianchi, D. Chillag, M. Lewis, and E. Pacifici, *Character degree graphs that are complete graphs*, Proc. Amer. Math. Soc. **135**, (2007), 671-676.
- J. Brough, *On vanishing criteria that control finite group structure*, J. Algebra, **458**, (2016), 207-215.

bibliography

bibliography

- D. Chillag and M. Herzog, *On the length of the conjugacy classes of finite groups*, J. Algebra, **131(1)**, (1990), 110-125.
- S. Dolfi, E. Pacifici, and L. Sanus, *Groups whose vanishing class sizes are not divisible by a given prime*, *Arch. Math. (Basel)*, **94** (2010) 311–317 .
- I. M. Isaacs, G. Navarro, and T. R. Wolf, *Finite group elements where no irreducible character vanishes*, *J. Algebra*, **222(2)**, (1999), 413-423.
- S. Mahmood Robati and R. Hafezieh, *Groups whose vanishing class sizes are prime powers*, *J. Group Theory*, **25(1)**, (2022), 151-162.
- S. Mahmood Robati and R. Hafezieh, *Groups whose vanishing class sizes are* p*-powers*, *Comm. in Algebra*, to appear.