Left 3-Engel elements in groups

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(joint work with G. Tracey and G. Traustason)

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Outline

Engel elements

- Let G be a group. We say that $g \in G$ is a right Engel element if for any $x \in G$, $\exists n = n(g, x) \geq 1$ such that $[g, {}_n x] = 1$, where

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- Similarly g is (bounded) left Engel if for any $x \in G$, $\exists n = n(g,x) \geq 1$ such that $[x, {}_n g] = 1$ ($\exists n = n(g) \geq 1$ such that $[x, {}_n g] = 1$).

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Relation between these sets: Heineken's results

- $R_n(G)^{-1} \subseteq L_{n+1}(G)$
- $R(G)^{-1} \subseteq L(G)$

Engel groups

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(Some) Facts:

- If G is locally nilpotent, then G is Engel.
- If G is of exponent 3, then G is 2-Engel. (Burnside)

Outline

1. Engel elements and groups
2. Known results
3. The “Gunnar” Lie algebra E
4. Our counterexample
5. Last, but not least

Small values of n

n-Engel groups

- G is 1-Engel $\iff$ G is abelian.
- G is 2-Engel $\implies$ G is nilpotent of class ≤ 3 (Burnside, Hopkins, Levi)
- G is 3-Engel $\implies$ G is locally nilpotent (Heineken)
- G is 4-Engel $\implies$ G is locally nilpotent (Havas and Vaughan-Lee)

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Right n-Engel elements

- x right 1-Engel $\iff x \in Z(G)$.
- x right 2-Engel $\implies x$ left 2-Engel. Right 2-Engel elements form a characteristic subgroup. (Kappe)
- x right 3-Engel $\implies \langle x \rangle^G$ nilpotent of class ≤ 3. (Newell)

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$\Longrightarrow g$ is left Engel. □

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Question (Abdollahi, 2010)

What is the least positive integer n for which there is a group G with $L_n(G) \not\subseteq \mathrm{HP}(G)$?

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Theorem (–, Traustason, Tracey)

Does the same hold for left 3-Engel elements? No!

Outline

Construction of E

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- We also add another distinct (formal) element, which is called x.
- The space L is then defined to be the infinite dimensional vector space over $\mathbb{F}$ spanned by the set

$$B = \{x\} \cup \{u_A : A \in F(\mathbb{N})\} \cup \{v_A : A \in F(\mathbb{N})\} \cup \{w_A : A \in F(\mathbb{N})\}.$$

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Lemma (–, Traustason, Tracey)

The associative enveloping algebra E is 12-dimensional.

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The costruction of the group G

Now we want to find a group G containing $1 + \mathrm{ad}(x)$ where $1 + \mathrm{ad}(x)$ is a left 3-Engel element in G but where $\langle 1 + \mathrm{ad}(x) \rangle^G$ is not nilpotent.

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- Since $a^2 = 1$ for all $a \in 1 + B$, the set G forms a group with identity element 1.

The costruction of the group G – II

We set

- $\mathcal{U} = \langle 1 + \mathrm{ad}(u_A) : A \subseteq \mathbb{N} \rangle$
- $\mathcal{V} = \langle 1 + \mathrm{ad}(v_A) : A \subseteq \mathbb{N} \rangle$
- $\mathcal{W} = \langle 1 + \mathrm{ad}(w_A) : A \subseteq \mathbb{N} \rangle$.
- Note that $\mathcal{U}, \mathcal{V}, \mathcal{W}$ are elementary abelian of countably infinite rank.
- We will be working with

$$G = \langle 1 + \mathrm{ad}(x), \mathcal{U}, \mathcal{V}, \mathcal{W} \rangle.$$

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Theorem (–, Traustason, Tracey)

The element $1 + ad(x)$ *is a left 3-Engel element in* G*.*
However $\langle 1 + ad(x) \rangle^G$ *is not nilpotent.*

Some remarks

- We now define some algebra E^* "from E".

Proposition (–, Traustason, Tracey)

We have $(1 + E^*)^{32} = 1$.

Proposition (–, Traustason, Tracey)

Any r*-generator subgroup of* $1 + E^*$ *is nilpotent of* r*-bounded class.*

Some remarks

- If we take any r conjugates $(1 + \mathrm{ad}(x))^{g_1}, \ldots, (1 + \mathrm{ad}(x))^{g_r}$ of $(1 + \mathrm{ad}(x))$ in G, they generate a nilpotent subgroup of r-bounded class that grows linearly with r. In particular:

Proposition (–, Traustason, Tracey)

Let $(1 + ad(x))^{g_1}, \ldots, (1 + ad(x))^{g_r}$ be any r conjugates of $1 + ad(x)$ in G. Then the group generated by these conjugates is nilpotent of class at most $4r + 2$.

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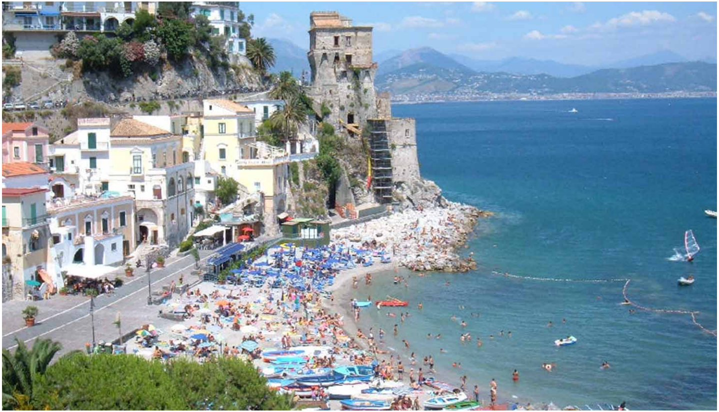
This is Cetara!



Cetara is a cozy fishermen's village nested along the Amalfi Coast among verdant citrus groves.

GTG 2019 in Cetara

Groups and Topological Groups 2019

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VI ASPETTIAMO!

Eskerrik asko!
Grazie :)

CS ;)