Large Characteristic Subgroups

Università degli Studi di Napoli Federico II

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Advances in Group Theory and Applications 2019

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The **trivial** subgroup in finite groups.

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Proposition *Let* G *be a group having a subgroup* H *of finite index , then* H *contains a normal subgroup of finite index.*

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Proposition *Let* G *be a group having a subgroup* H *of index* n*, then* H *contains a normal subgroup of index at most* $n!$.

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Definition A subgroup H of a group G is said **characteristic** when $\alpha(H) = H$ for all $\alpha \in \mathrm{Aut}(G)$.

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$$\mathsf{K}\ \textit{char}\ \mathsf{H} \lhd \mathsf{G} \implies \mathsf{K} \lhd \mathsf{G}$$

- If A is an **abelian** subgroup of finite index in an infinite normal subgroup H of a group G, does G have a normal non-trivial **abelian** subgroup?

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• Is abelianity F-**characteristic?**

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Is it possible to find a **characteristic** subgroup of G having **finite index** and satisfying the same **properties** of H? ...and maybe even get a bound for the index?

Set $n = |G : H|$.

1979 – 1972 Abelianity

Let G be a group with an abelian subgroup A of finite index, then G contains a characteristic abelian subgroup of finite index.

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bound – n^n

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2008 Abelianity

*Let G be a **finite** group with an abelian subgroup A of index n, then G contains a characteristic abelian subgroup of index at most n^2.*

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2008 Abelianity

Let G be a **finite** *group with an abelian subgroup A of index n, then G contains a characteristic abelian subgroup of index at most n^2.*

2017 Abelianity

Let G be **any** *group with an abelian subgroup A of index n, then G contains a characteristic abelian subgroup of index at most n^2.*

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1938 Nilpotency

Let G be a group with a **nilpotent** *subgroup of finite index, then G contains a characteristic nilpotent subgroup of finite index.*

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Nilpotency

2004 Bruno & Napolitani *Let G be a group with a nilpotent subgroup of finite index having class c. Then G has a characteristic nilpotent subgroup of finite index and of class at most c.*

bound – $c = 2: \ (n^{2n})$

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2018 *Let $\mathfrak{X}$ be an F-characteristic group class which is S_n, H and R_0-closed. Then the class of all* **central-by-$\mathfrak{X}$** *groups is F-characteristic.*

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2007 Solubility *Let G be a group having a soluble subgroup of finite index and with defect d. Then G contains a characteristic soluble subgroup of finite index having defect at most d.*

bound $2^{f^{2^d - 1}(\log_2(n!))}$

$f(x) = x(x+1)$

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2007 Verbal properties *Let θ be an outer commutator word. Then the variety $\mathcal{W}(\theta)$ is an F-characteristic group class.*

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A union of F-characteristic group classes is F-characteristic.

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2018 Corollary
The class of groups with a nilpotent commutator subgroup is F-characteristic.

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The class of groups with a **locally nilpotent** *commutator subgroup is* F*-characteristic.*

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- *The class of Baer groups and the class of Gruenberg groups*

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- *The classes of finite-by-abelian groups, finite-by-(nilpotent of bounded class) groups, ...*

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– Swan *Any torsion-free group containing a free subgroup of finite index is likewise free*

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- *The class of free (abelian) groups*
- *The class of torsion-free (abelian) groups*
- *The class of non-trivial simple groups*

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The class of **periodic quasihamiltonian** *groups is* F*-characteristic.*

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Definition
A group G *is* **quasihamiltonian** *whenever* $HK = KH$ *for each* $H, K \leqslant G$.

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2018 Intersection
The intersection of two F*-characteristic group classes closed by taking subnormal subgroups is* F*-characteristic*

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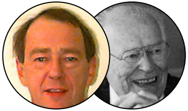

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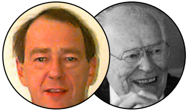

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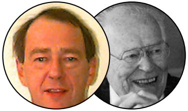

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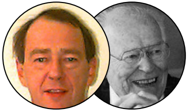

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- *Finitely generated soluble* T*-groups are either finite or abelian.*
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2018
If G *is any soluble* T*-group, then any subgroup containing the* $Fit(G) = C_G(G')$ *is characteristic in* G*.*

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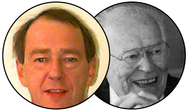
The class of **periodic** *soluble* T*-groups is* F*-characteristic.*



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If G *is a group having a subgroup of finite index which is a periodic* T*-group, then it also contains a characteristic subgroup of finite index which is* $\overline{T}$*-group.*

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Corollary

The class of soluble $\overline{\mathsf{T}}$-groups is F-characteristic.

Non-periodic soluble $\mathfrak{T}$-groups

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- Let X be a subgroup of finite index of a soluble T-group of **type** 2, and let T be its periodic part. Then T' is characteristic in G.

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- Let X be a subgroup of finite index of a soluble T-group of **type** 2. Then X is also a T-group of type 2 and $X' = G'$.

Theorem
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Example
There exists a metabelian group containing a subgroup of finite index which is a T-group of **type** 2 but no characteristic subgroup of finite index with the T-property.

Thank you all!

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Università degli Studi di Napoli Federico II

Marco Trombetti

Advances in Group Theory and Applications 2019

Lecce — 26 June 2019