A property of the Lamplighter group

joint work with G. Corob Cook and P.H. Kropholler

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Advances in Group Theory and Applications

Lecce, June 2019

The Lamplighter Group

Definition

The Lamplighter group is defined to be the standard restricted wreath product $\mathbb{F}_2 \,\mathrm{wr}\, \mathbb{Z}$, i.e, it is the semidirect product

$$\bigoplus_{\mathbb{Z}} \mathbb{F}_2 \rtimes \mathbb{Z}$$

where $\mathbb{Z}$ acts on the direct sum by shifting the components.

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The Lamplighter group admits the following minimal presentation

$$\langle\, \sigma, \tau \mid \sigma^2,\, [\tau^k \sigma \tau^{-k}, \tau^\ell \sigma \tau^{-\ell}], \quad \forall k, \ell \in \mathbb{Z} \,\rangle.$$

Therefore, every element X has the normal form

$$x = (\tau^{k_1} \sigma \tau^{-k_1})(\tau^{k_2} \sigma \tau^{-k_2}) \cdots (\tau^{k_n} \sigma \tau^{-k_n}) \tau^\ell \quad \text{with } k_1 \leq k_2 \leq \cdots \leq k_n.$$

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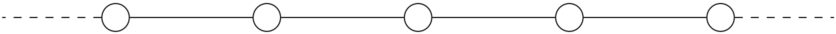
with streetlamps in either direction



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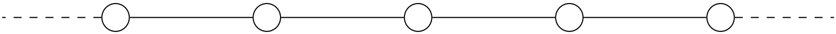
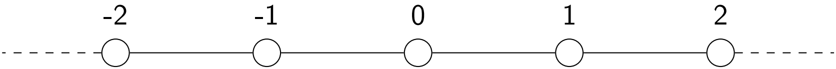


indexed by the integers in $\mathbb{Z}$.



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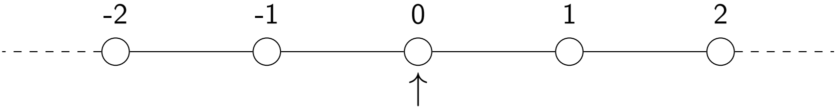

Rules of the Dynamical System:

1. The lamplighter stands at one lamp.

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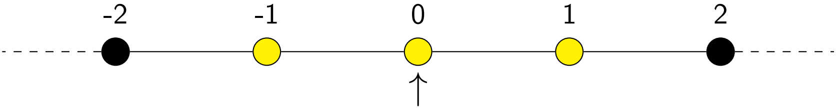

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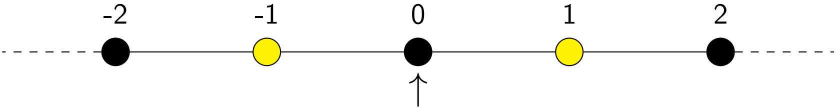

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1. The lamplighter stands at one lamp.
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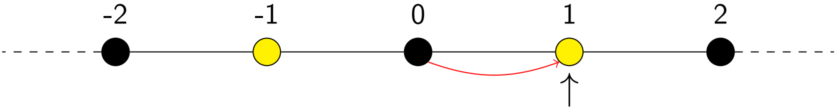

Rules of the Dynamical System:

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2. A finite number of lamps are illuminated ● (yellow) while all others remain off ●.
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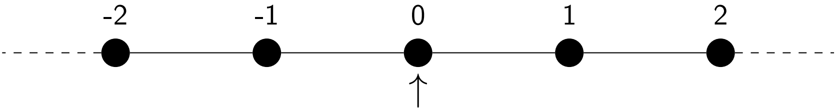
4. The lamplighter can move one step to his right/left.

The dynamics of the Lamplighter

Let ε be the configuration in which all lamps are off and the lamplighter stands at index 0



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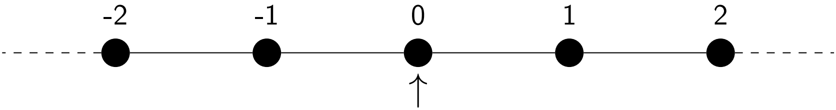

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Fact: The Lamplighter group consists of all possible configurations that one can obtain from the empty lampstand by performing a finite number of actions 3 and 4.

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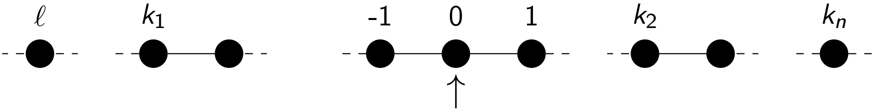

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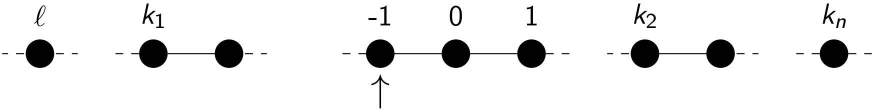

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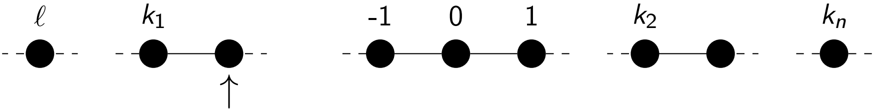

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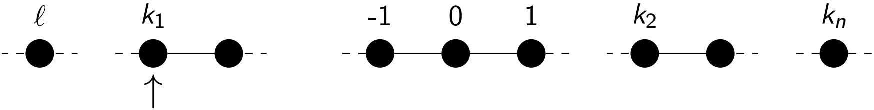

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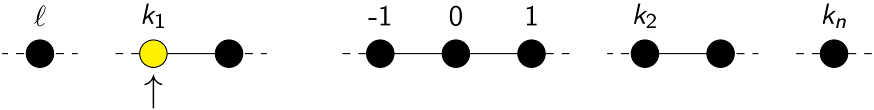

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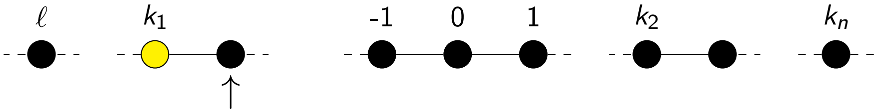

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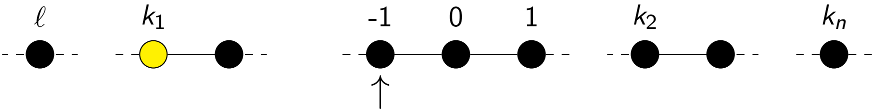

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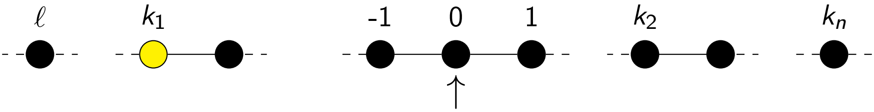

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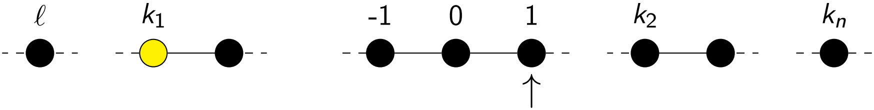

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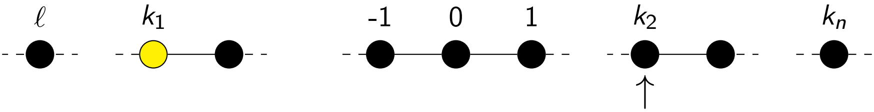

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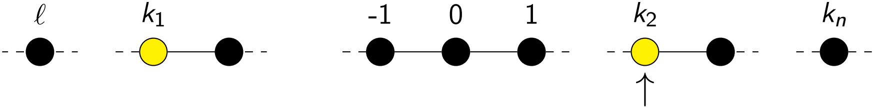

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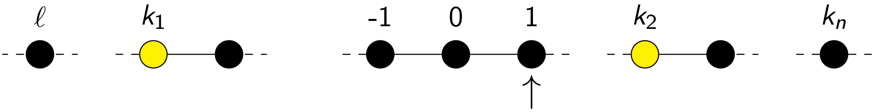

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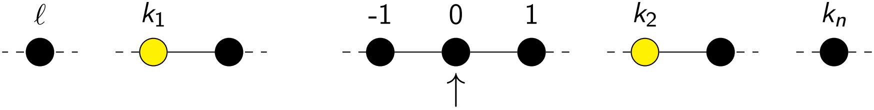

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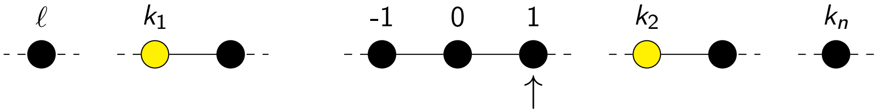

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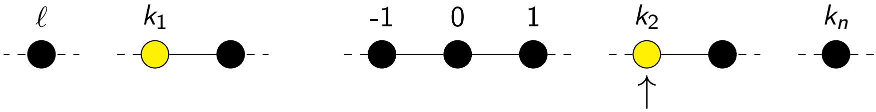

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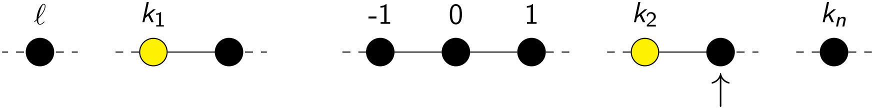

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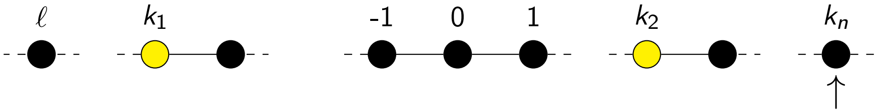

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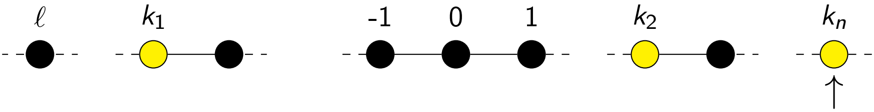

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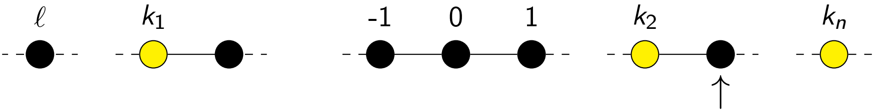

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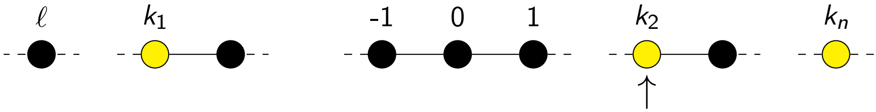

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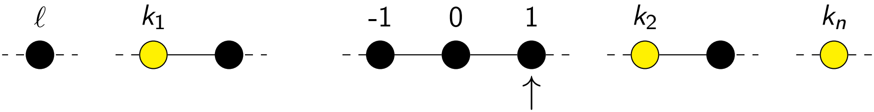

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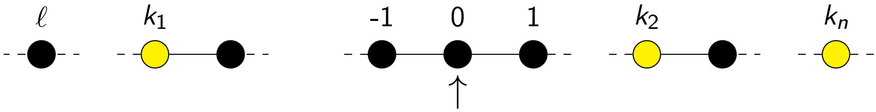

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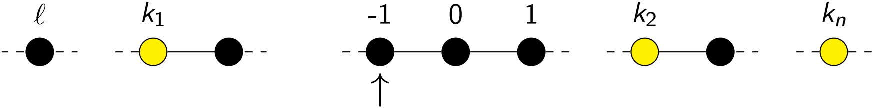

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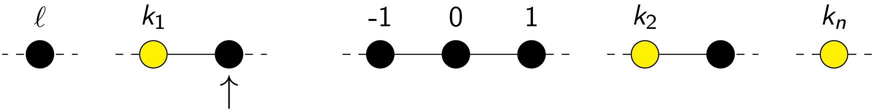

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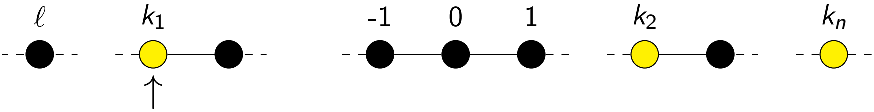
- $\sigma =$ the Lamplighter switch the lamp where he is standing.

Every element x of the Lamplighter group admits a normal form

$$x = (\tau^{k_1}\sigma\tau^{-k_1})(\tau^{k_2}\sigma\tau^{-k_2})\cdots(\tau^{k_n}\sigma\tau^{-k_n})\tau^{\ell}$$

for some $k_1 \leq k_2 \leq \cdots \leq k_n$.



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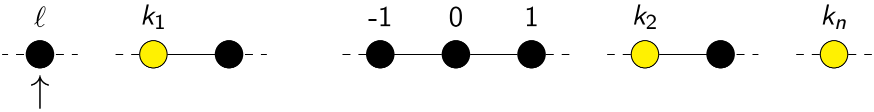

Inert subgroups

Definition

A subgroup H of a group G is said to be *inert* (or *commensurated*) if H and $H^g = gHg^{-1}$ are commensurate for all $g \in G$, meaning that $H \cap H^g$ always has finite index in both H and H^g.

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Theorem (C.–Corob Cook–Kropholler, 20??)

The inert subgroups of the lamplighter group $\mathbb{F}_p \operatorname{wr} \mathbb{Z}$ fall into exactly five classes.

The base $B = \bigoplus \mathbb{F}_p \cong \mathbb{F}_p[x, x^{-1}]$

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Let G be a group, $\mathbb{K}$ a field and $\mathbb{K}G$ the group algebra.

Definition

A $\mathbb{K}-$subspace U of a $\mathbb{K}G-$module V is *G-almost invariant* when $U/U \cap Ug$ is finite dimensional for all $g \in G$.
Subspaces U and W are *almost equal* when $U/U \cap W$ and $W/U \cap W$ are both finite dimensional.

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Let H be an inert subgroup of the lamplighter group. The subspace $H \cap B$ is a $\langle x \rangle$-almost invariant subspace of the base B.

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Lemma

Let $B^+ = \mathbb{F}_p[x]$ and $B^- = \mathbb{F}_p[x^{-1}]$. If U is a $\langle x \rangle$-almost invariant subspace of B, then U is almost equal to one of the four subspaces $0, B^+, B^-, B$.

Connection to topological groups

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Proposition (van Dantzig's, 1930s)

A topological group $\bar{G}$ is totally disconnected and locally compact (TDLC, for short) if, and only if, the family $\mathcal{CO}(\bar{G})$ of all compact open subgroups of $\bar{G}$ is a local basis at the identity.

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All compact open subgroups in $\bar{G}$ are commensurate with one another and therefore inert.

Connection to topological groups

Corollary (C.–Corob Cook–Kropholler, 20??)

If $\bar{G}$ is a totally disconnected locally compact group which has a dense subgroup isomorphic to a lamplighter group then $\bar{G}$ is isomorphic to one of the following.

1. *A discrete lamplighter group.*
2. *A compact group.*
3. *The group $\mathbb{F}_p((t)) \rtimes \mathbb{Z}$ for some prime p.*
4. *The unrestricted wreath product $\mathbb{F}_p \,\overline{\mathrm{wr}}\, \mathbb{Z}$ for some prime p.*

Connection to algebraic entropy

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Let A be and abelian group and $\alpha\colon A \to A$ an endomorphism. Denote by $\mathcal{F}(A)$ the set of all finite subgroups of A.

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Let A be and abelian group and $\alpha\colon A \to A$ an endomorphism. Denote by $\mathcal{F}(A)$ the set of all finite subgroups of A.

$$T_n(\alpha, F) := F + \alpha(F) + \cdots + \alpha^{n-1}(F),$$

denotes the *partial trajectory in* F;

$$\mathrm{ent}(\alpha, F) := \lim_n \log[T_n(\alpha, F)\colon F],$$

defines the *partial entropy of* α *in* F;

$$\mathrm{ent}(\alpha) := \sup\{\mathrm{ent}(\alpha, F) \mid F \in \mathcal{F}(A)\}$$

is the algebraic entropy of α.

Connection to algebraic entropy

Definition (Dikranjan–Giordano Bruno–Salce–Virili, 2015)

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$$\widetilde{\mathrm{ent}}(\alpha) := \sup\{\mathrm{ent}(\alpha, I) \mid I \in \mathcal{I}_\alpha(A)\}$$

is the *intrinsic algebraic entropy of α.*

Thanks for your attention