① G finite acts freely on sphere

$\Downarrow$

② $0 \to \mathbb{Z} \xrightarrow{i} P_{q-1} \to \cdots \to P_0 \xrightarrow{\varepsilon} \mathbb{Z} \to 0$

P_i $\mathbb{Z}G$-mod

$\Updownarrow$

③ $H^i(G, -) \simeq H^{i+q}(G, -)$ $\forall i > 0$

$(\hat{H}^i(G, -) \simeq \hat{H}^{i+q}(G, -)$ $\forall i \in \mathbb{Z})$

Tate coh

$\Downarrow$ Swan

$\exists$ finite s.c. $X \simeq$ sphere G

freely & simplicially on X

$H^i(G_p, -) \simeq H^{i+2}(G_p, -) \quad \forall\, i \geq 1$

G freely & properly on $\mathbb{R}^n \times S^{k}$

$\Rightarrow$ G periodic com after n-step

Conj The f.a.e for G

1) $\exists$ a finite dim free G-CW comp. $X \simeq S^m$

$\Downarrow$ $\quad$ ||| G countable

G acts freely & prop on $\mathbb{R}^n \times S^m$

2) $0 \to \mathbb{Z} \to A \to P_{q-2} \to \cdots \to P_0 \to \mathbb{Z} \to 0$

P_i p.d$_{\mathbb{Z}G} A < \infty$ $\quad \mathrm{Ext}^q_{\mathbb{Z}G}(\mathbb{Z}, \mathbb{Z})$

$\Downarrow$

3) $H^i(G, -) \simeq H^{i+q}(G, -)$ $\quad H^q(G, \mathbb{Z}G)$

$\forall i > f$.

T. Johnson / Connolly-Prassidis

$\mathrm{vcd} < \infty$

Mislin + T

$G \in \mathrm{H}\mathfrak{F}_b$

$(1) \Leftrightarrow (2) \Leftarrow (3)$

$\Downarrow$

$G \in \mathrm{H}_1\mathfrak{F}_b$

$(3) = (4)$

(4) every finite subgp has periodic coh.

$\mathrm{vcd}\, G < \infty \subsetneq H, fb$

$B(n,e)$. e odd exp. ≥ 665

Ivanov

finite Tate co

vcd G < ∞ Farrell - Tate cohom.

C.T.C. Wall

If G admits c.r. $\exists\, n$:

$$H^n(G, \text{proj}) \neq 0$$

$$n \in \mathbb{Z}$$

$$\begin{array}{ccccccccccc}
\underline{\underline{E}}: & \cdots & \to & E_n & \to & E_{n-1} & \to & \cdots & & & \\
\| & \| & & \| & & & & & & & \\
 & \cdots & \to & P_n & \to & P_{n-1} & \to \cdots \to & P_0 & \to & M & \to 0
\end{array}$$

$$\widehat{\mathrm{Ext}}^i_R(M,N) = H^i(\mathrm{Hom}_R(\underline{\underline{E}}, N))$$

$$i \in \mathbb{Z}$$

$\mathrm{Hom}_{\cancel{R}}(\underline{\underline{E}}, \mathrm{proj})$ acyclic $\forall$ proj

Adem-Smith

(2) $\Leftrightarrow$ G f $\leftarrow \mathbb{R}^n \times S^m$ $\forall G$

<u>Thm</u>. The f.a.e. G with period

whom after some step

4) $0 \to \mathbb{Z} \to A \to P_{q-2} \to \cdots \to P_0 \to \mathbb{Z} \to 0$

$P_{\frac{d}{2}}(A < \infty$

2) $G \curvearrowright_{\mathbb{Z}} G < \infty$

Example of Gor. proj

1). Every proj. is Gor. proj

2) G finite every $\mathbb{Z}$-free $\mathbb{Z}G$-mod is Gor proj

3) $H \leq G$ M Gor pr $\mathbb{Z}H$-mod then $\Rightarrow$ $\mathbb{Z}G \otimes M$ Gor. proj. $\mathbb{Z}G$ $\mathbb{Z}H$

4) $\oplus$, $\varinjlim$

5) $G \in HF$ Gorenst. proj $=$ cofibrant mod Benson

Gor. p. dim

G. dim.
Asgslander - Briolope

Fruchs - Jemla

Asadollahi - Bahlereh
- Salarian

Properties of $Gcd\, G$

1) If $cd_{\mathbb{Z}} G < \infty$ $\quad cd_{\mathbb{Z}} G = Gcd_{\mathbb{Z}} G$

($Gcd_{\mathbb{Z}} G = 0 \Leftarrow |G|$ finite $\quad cd_{\mathbb{Z}} G = \infty$

2) If $vcd_{\mathbb{Z}} G < \infty \longrightarrow vcd_{\mathbb{Z}} G = Gcd_{\mathbb{Z}} G$.

$$G = A \underset{H}{*} B$$

$$1 \to \langle x \rangle \to G \to K \to 1$$

$$cd_{\mathbb{Z}} K < \infty$$

Bahletch-Rombeguir + T

3) $H \leq G$ $\quad \mathrm{Gcd}_{\mathbb{Z}} H \leq \mathrm{Gcd}_{\mathbb{Z}} G$

4) $1 \to N \to G \to K \to 1$

$$\mathrm{Gcd}_{\mathbb{Z}} G \leq \mathrm{Gcd}_{\mathbb{Z}} N + \mathrm{Gcd}_{\mathbb{Z}} K.$$

5) If $\mathrm{Gcd}_{\mathbb{Z}} G < \infty$ $\quad F \leq G \quad |F| < \infty$

$$\mathrm{Gcd}_{\mathbb{Z}} N_G(F)/F \leq \mathrm{Gcd}_{\mathbb{Z}} G$$

6) $\mathrm{Gcd}_{\mathbb{Z}} G \leq 1 \implies 0 \to \mathbb{Z} \to A$

$\Updownarrow$?C $\qquad\qquad \mathrm{p.d}_{\mathbb{Z}G} A \leq 1$

$\Rightarrow$ G acts on a tree with finite v.s

- If $\mathrm{cd}_k G < \infty \longrightarrow \mathrm{cd}_k G \leq \mathrm{Gcd}_{\mathbb{Z}} G$

- $\mathrm{Gcd}_{\mathbb{Z}} G = \sup\{n : \mathrm{Ext}^n_{\mathbb{Z}G}(M, \mathrm{proj}) \neq 0\}$
  M $\mathbb{Z}G$-mod $\mathbb{Z}$-free

- If $\mathrm{Gcd}_{\mathbb{Z}} G < \infty$ then Holm

$$\mathrm{Gcd}_{\mathbb{Z}} G = \sup\{n : H^n(G, \mathrm{proj}) \neq 0\}$$

G countable (Emmanouil + T)

$$\mathrm{Gcd}_{\mathbb{Z}} G = \sup\{n : H^n(G, \mathbb{Z}G) \neq 0\}$$

$\mathrm{Ext}^n_R(A,B)=0$ A, B, R countable

$\Downarrow$

$\mathrm{Ext}^n_R(A, \bigoplus_S B)=0$

$\mathrm{Gcd}_2 G < \alpha$ iff G admits a complete resol in the s.S

Conj.

$$\mathrm{Gcd}_{\mathbb{Z}} G < \infty \iff G \in H_1 F$$

$$\| $$

$$\{ G : \exists \text{ fin. dim. contr. } G\text{-CW-cx with finite stab.} \}$$

$$\Longleftarrow$$

$$0 \to \bigoplus \mathbb{Z}(G/G_{i_n}) \to \dots \to \bigoplus \mathbb{Z}(G/G_{i_0}) \to \mathbb{Z} \to 0$$

$$\bigoplus \mathbb{Z}(G/G_{i_n}) \cong \bigoplus \mathbb{Z}G \otimes_{\mathbb{Z}G_{i_n}} \mathbb{Z}$$

Conj

if G has a c.r.

$$\Downarrow$$

G has a c.r in the s.s

$$\Updownarrow$$

$$\mathrm{cd}_{\mathbb{Z}} G < \infty$$

if $H^i(G,-) \cong H^{i+q}(G,-)$ $\forall i > k$

$$\Downarrow$$

$$\begin{tikzcd}
R_k \arrow[r] & P_{k+q-1} \arrow[r] & \cdots \arrow[r] & P_k \arrow[r] \arrow[d] & R_k. \\
 & & & R_k &
\end{tikzcd}$$

$\Longrightarrow$

$G \in H\mathfrak{F}_b$ Krop. Mislin

$G \in H\mathfrak{F} \cap FP_\infty \longrightarrow G \in H_1\mathfrak{F}$

$\Downarrow$ $\Nearrow$

$G \in H\mathfrak{F}_b \cap \mathrm{p.d}_{\mathbb{Z}G} B(G,\mathbb{Z})$

$\mathrm{Gcd}_{\mathbb{Z}} G$

Lück.

G s.t. $l(G) < \alpha$ & $B(n)$

$\longrightarrow G \in \mathcal{H}_1\mathcal{F}$

$G \quad B(n) \Rightarrow G \quad B(n_0)$

$n_0 \leq n$

$\mathrm{Gal}_{\mathbb{C}}$