

Tarski Monsters

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The existence of Tarski groups
was proved by A.I. Olshanskiĭ, who constructed
an infinite simple p -group of this type
for each sufficiently large prime number p



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Notice also that Tarski groups have rank 2



Let k be a positive integer
We say that a group G is a *Tarski k -monster* if

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Every Tarski k -monster has finite rank, either k or $k + 1$



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Ol'shanskii's methods can be used to show
that for each k there exists a Tarski k -monster



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A positive answer to this question
would show that any infinite group
satisfying both *min* and *max*
is a Tarski k -monster for some positive integer k

