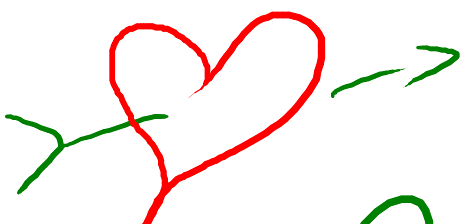

OPEN

Problem 1

Session

G Dalla Volta + Mills ~ 1951

regular subgroups
$$N \leq S(G)$$
which have the same holomorph as G

$$N_{S(G)}(N) = N_{S(G)}(\rho(G))$$

$(G,+)$ f.g. abelian group

translates to

what are the rings $(G, +, \cdot)$

s.t. $\boxed{\mathrm{Aut}(G,+) \leq \mathrm{Aut}(G,+,\cdot)}$

study this condition for other classes of abelian groups

Def: A finite group G is IYB group if it is the mult. group of a left brace.

Thm (ESS) An IYB group is solvable

Thm (Bachmuller 2016)

There exists n s.t. for every prime $p > n$ there exists a finite p-group of class 9 which is not IYB

$$G = \langle a_1, \ldots, a_n \mid [a_j, a_i]^{p^{\alpha_{ji}}} = [[a_j, a_i], a_k]^{p^{\beta_{jik}}} = 1, \; [[a_j, a_i], a_k] \text{ central} \rangle$$

$G/Z(G)$ is finite $\Rightarrow$ G' fte $\forall\, G$

(Schur)

$G/Z_k(G)$ is finite $\Rightarrow$ $\gamma_{k+1}(G)$ fte $\forall\, G$

(Baer)

$G/Z_\infty(G)$ is fte $\Rightarrow$ $\exists\, L \lhd G$, L fte

s.t. G/L is hypercentral

G generalized radical

$G/Z_k(G)$ finite rank $\Rightarrow$ $\gamma_{k+1}(G)$ fte rank

Q^n: Suppose G is generalized radical and suppose $G/Z_\infty(G)$ has finite 0-rank.. Does $\exists$ $L \lhd G$ such that L has finite 0-rank & $G/L \in L\mathfrak{N}$?

Kaplansky conj

G t.f. gr Γ

K field

$U(KG) = \{ kg \}$
$k \neq 0$

$\Gamma : X \times X \longrightarrow X \times X$

yB $(x, y) \longmapsto (\sigma_x(y), \gamma_y(x))$

$G = \langle x \mid x \in X,$

$xy = \sigma_x(y)\gamma_y(x) \rangle$

t.f., kG domain

(G abelian-by-finite)

$$
\begin{array}{c}
G \hookrightarrow Fa_n \times S_n \\
\big\downarrow \\
\overline{Fa_n}
\end{array}
$$

$$\langle x_1, x_2, x_3, x_4 \mid \ \rangle \longrightarrow$$

$$\langle x, y \mid x^{-1}y^2x = y^{-2}, y^{-1}x^2y = x^{-2} \rangle$$

Find more classes of finite simple left braces

1) Construct new classes of left braces $(B, +, \cdot)$ of order p^n, p a prime.

2) Describe $Aut(B, +, \cdot)$ of all such braces

If B a bat tree

$$B = B_1 B_2 \dots B_n$$

B_i sylow subgps of $(B, +)$

B_i is a subtree

3) $p \neq q$ primes

Describe all simple left braces of order $p^n q^m$

(ZC) Every augmentation 1 torsion element of $\mathcal{U}(\mathbb{Z}G)$ is conjugate in $\mathbb{Q}G$ to an element in G.

(SP) ?

$$\mathrm{Spec}(V(\mathbb{Z}G)) = \mathrm{Spec}(G)$$

$$ZC \Rightarrow SP$$

p q

G has an element of order pq

Prime Graph of $V(\mathbb{Z}G)$ = Prime Graph of G (PGQ)

$p = |g|$

$g \in G$

$$(ZC) \Rightarrow (GBP) \Rightarrow (SP) \Rightarrow (PGQ)$$

$$a = \sum_{g \in G} a_g g, \quad X \subseteq G$$

$$\varepsilon_X(a) = \sum_{x \in X} a_x$$

$$G[n] = \{ g \in G : |g| = n \}$$

Generalized Bovdi: torsion unit of of order n

$$\varepsilon_{G[m]}(u) = 0 \; ?$$

$m \neq n$

(GBP)

(PAP) $\quad$ u torsion element in $V(\mathbb{Z}G)$

$C \in Cl(G)$

$$\varepsilon_C(u^n) = \sum_{\substack{D \in Cl(G) \\ g \,:\, g^n \in C}} \varepsilon_D(u)$$

$$u \longrightarrow 1$$
$$\mathbb{Z}G \longrightarrow \mathbb{Z}(G/N)$$

N nilp. normal subgroup

PAP holds

$$(ZC) \Longrightarrow (PAP) \Longrightarrow (GBP)$$
$$\Downarrow$$
Positive: G solvable $\Longleftarrow$ (SP)
$$\Downarrow$$
$$(PG)$$

1. If a group G admits a complete resol. then it admits a r.r. in the s.s

If G admits a c.r

$\Downarrow$

fin.dim $\mathbb{Z}G < \infty$

2. If G is a countable soluble group then there is at most one n s.t

$$H^n(G, \mathbb{Z}G) \neq 0$$

F field

A $\mathbb{N}_0$-grad F-alg

$\eta : T(A_1) \longrightarrow A$

η surj $\iff$ A 1-gen

η surj

$\ker(\eta) = \langle \ker(\eta_2) \rangle$

$\iff A$ quad

$\dim A_1 < \infty$, A quad

$$A^! = T(A_1^*) \Big/ \left(\ker(\eta_2)^{\perp} \right)$$

G pro-p (f.g.)

$$A = \mathbb{F}_p[[G]] = \varprojlim_n \mathbb{F}_p[G/n]$$

$$W = \ker(\mathbb{F}_p[G] \to \mathbb{F}_p)$$

$$gr(G) = \coprod W^k / W^{k+1}$$

$$J_k(G) = G \cap 1 + w^k$$

(Zassenhaus)

$$gr(G) = \underline{u}(L(G))$$

Lie w. 2.

T.f.a.e. (G prop)

G uniformly p.f (rk r)

$$\mathrm{gr}(G) = \overline{\mathbb{F}}_p[x_1, \dots, x_n]$$

$$H^{*}(G, \overline{\mathbb{F}}_p) = \Lambda[\bar{x}_1^{*}, \dots, x_n^{*}]$$

$\parallel$

$n(G)$

$$gr(G)^{!\,op} = h(G)$$

$$gr(G) = h(G)^{!}$$

(koszul prop.)

<u>Q</u>: G pro-p (f.g.)

s.th. $h(U)$ quad $\forall U \leq_o G$

$$\overset{?}{\Longrightarrow} \quad gr(G) = h(G) \,!$$

Gr.

$$\Rightarrow \quad gr(G_F(p)) \overset{\text{\"o}}{=} K^M_\bullet(F)/p \qquad F \text{ field}, \ldots$$

$$G_F(p) = G_F / O^p(G_F)$$