f-subnormality and Wielandt-like subgroups in Infinite Groups

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Absolute properties

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Let θ be a property pertaining to subgroups of a group

θ is called **absolute** if in any group G all subgroups isomorphic to some θ-subgroup are likewise θ-subgroups

θ is absolute if and only if there exists a group class $\mathfrak{X} = \mathfrak{X}(\theta)$ such that in any group G a subgroup H has the property θ if and only if H belongs to $\mathfrak{X}$

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If θ is an embedding property for subgroups, a group class $\mathfrak{X}$ is said to **control** θ if it satisfies the following condition:

• if G is any group containing some $\mathfrak{X}$-subgroup, and all $\mathfrak{X}$-subgroups of G have the property θ, then θ holds for all subgroups of G

Examples

The class of finitely generated groups controls every local property

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W. Möhres in 1990 shows that every group in which all subgroups are subnormal is soluble

Large groups

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- $\mathfrak{X}$ contains no finite cyclic groups

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However, the locally dihedral 2-group shows that normality cannot be controlled by the class $\mathfrak{I}$

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...and hence groups of **infinite rank** form a class of large groups

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The class $\mathfrak{X}$ has been defined by N.S. Černikov in 1990

A positive result

Theorem (M.J. Evans and Y. Kim - 2004)

If G is an $\mathfrak{X}$-group of infinite rank and each subgroup of infinite rank of G is subnormal of defect at most d, then G is nilpotent of class at most $g(d)$, for some function g.

(n, m)-subnormal subgroups

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Every subgroup of a finite group G is $(n, 0)$-subnormal in G, for some $n \leq |G|$

(n, m)-subnormal subgroups

J. C. Lennox:
"On groups in which every subgroup is almost subnormal", *J. London Math. Soc.* (2) **15** (1977), no. 2, 221-231

"Joins of almost subnormal subgroups", *Proc. Edinburgh Math. Soc.* (2) **22** (1979), no. 1, 33-34

C. Casolo and M. Mainardis:
"Groups in which every subgroup is f-subnormal", *J. Group Theory* **4** (2001), no. 3, 341-365

"Groups with all subgroups f-subnormal", *Topics in infinite groups*, Quad. Mat., vol. 8, Dept. Math., Seconda Univ. Napoli, Caserta, 2001, pp. 77-86

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M. De Falco, F. de Giovanni, and C. Musella:
"Groups with normality conditions for subgroups of infinite rank", *Publ. Mat.* **58** (2014), no. 2, 331-340

(n, m)-subnormal subgroups

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Then every subgroup of G is of near defect at most (n, m), where $n = |N|$ and m is the nilpotency class of G/N

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Theorem (J.C. Lennox - 1977)

Let G be a group and let m, n be non-negative integers. Suppose that every finitely generated subgroup of G is of near defect at most (n, m). Then there is a function μ such that $|\gamma_{\mu(m+n)}(G)| \leq n!$.

Why the bounds?

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FC-groups G have the property that each finitely generated subgroup of G is of finite index in its normal closure, but

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Theorem (C. Casolo and M. Mainardis - 2001)

If G is a group in which every subgroup has finite index in a subnormal subgroup, then G is finite-by-soluble.

The group class $S(n, m)$

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L.A. Kurdachenko and H. Smith: "Groups in which all subgroups of infinite rank are subnormal", *Glasg. Math. J.* (2) **46** (2004), no.1, 83-89

L.A. Kurdachenko and P. Soules: "Groups with all non-subnormal subgroups of finite rank", *Groups St. Andrews* 2001 in Oxford. Vol. II, London Math. Soc. Lecture Note Ser., vol. 305, Cambridge Univ. Press, Cambridge, 2003, pp. 366-376

M.J. Evans and Y. Kim: "On groups in which every subgroup of infinite rank is subnormal of bounded defect", *Comm. Algebra* **32** (2004), no.7, 2547-2557

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Let G be an $\mathfrak{X}$-group in the class $S(n,m)$. Then G is soluble-by-finite.

Lemma

Let G be a group and suppose that A, B are (n,m)-subnormal subgroups of G. Then $A \cap B$ is an (n^2,m)-subnormal subgroup of G.

The easy cases

Theorem

Let G be a periodic $\mathfrak{X}$-group in the class $S(n, m)$. Then G is finite-by-nilpotent.

Lemma

Let G be a locally nilpotent $S(n, m)$-group. Then there is a function f such that G is nilpotent of class at most $f(n, m)$.

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Corollary

Let G be an $\mathfrak{X}$-group in the class $S(n, m)$. Then the Baer radical of G is nilpotent.

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Theorem

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Lemma

Let G be an $\mathfrak{X}$-group in the class $S(n,m)$ and suppose that the torsion subgroup T of the Baer subgroup B of G has infinite rank. If G is not finite-by-nilpotent, then T contains a primary component of infinite rank.

The other half

Lemma

Let G be an $\mathfrak{X}$-group in the class $S(n, m)$ and let A be a normal elementary abelian p-subgroup of G of infinite rank. Then G is finite-by-nilpotent.

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Theorem

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f-subnormality

Definition (R.E. Phillips - 1972)

A subgroup H of a group G is said to be f-**subnormal** in G if there is a finite chain

$$H_0 = H \leq H_1 \leq \ldots \leq H_{n-1} \leq H_n = G$$

such that $|H_i : H_{i-1}|$ is finite or H_{i-1} is normal in H_i for every $i \in \{1, 2, \ldots, n\}$.

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Theorem (C. Casolo and M. Mainardis - 2001)

A group in which each subgroup is f-subnormal is finite-by-soluble.

The f-Wielandt subgroup

Definition (H. Wielandt - 1958)

The **Wielandt subgroup** of a group G, $w(G)$, is defined to be the intersection of all normalizers of subnormal subgroups of G.

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The **norm** of a group G, $N(G)$, is the intersection of the normalizers of all its subgroups.

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$Z(G) \leq N(G) \leq \bar{w}(G) \leq w(G)$

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Proposition (M.R. Dixon and M.F. - 2017)

Let G be a group satisfying the minimal condition on subnormal subgroups. Then G satisfies the minimal condition on f-subnormal subgroups too.

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Lemma (M.R. Dixon and M.F. - 2017)

Let G be a group satisfying the minimal condition on subnormal subgroups and suppose that H is f-subnormal subgroup of G. Then H has only finitely many conjugates in G.

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Theorem (M.R. Dixon and M.F. - 2017)

Let G be a residually finite group. Then $\bar{w}(G) \leq Z_2(G)$. In particular $\bar{w}(G)$ is a Dedekind group.

Theorem (M.R. Dixon and M.F. - 2017)

Let G be a torsion-free residually finite group. Then $\bar{w}(G)$ is abelian.

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Theorem (M.R. Dixon and M.F. - 2017)

Let G be a torsion-free polycyclic group. Then $\bar{w}(G) = Z(G)$.

Thanks for your attention!