



## In Happy Memory of the Twenty Three Fruitful and Active Semesters Spent in Frankfurt <sup>1</sup>

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(*Communicated by Francesco de Giovanni* <sup>2</sup>)

It is said that old mathematicians never die, they only dissolve — into hot air presumably. So according to this the mathematician has no cause to bid a formal farewell.

But it is different when one not only experiences the enjoyment of research but also, as I do today, looks back on a decade of broad impact. We had the good fortune of attracting students who will continue where we left off and who perhaps will carry on in the same spirit.

The moment has therefore come to reflect on what we consider Mathematics to be and wherein lies the task of the mathematician. Of course we are aware that such personal perceptions may exist only in the realms of wishful thinking — ones which we wished to achieve and those of today's reality. For we naturally wish to see in ourselves the ideal mathematician, one concerned with “true” mathematics, one who tackles “real” problems and strives to make our ideal become a reality.

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<sup>1</sup> This is the reproduction of a talk given by Baer on his last lecture in 1967, before his retirement from the University of Frankfurt

<sup>2</sup> I am deeply grateful to Bernhard Amberg, who provided the original text, that was handed out by Baer to the participants afterwords in printed form, and to Martin Newell, who translated it into English. The original German text can be found at: [www.advgrouptheory.com/GTArchivum/Baer/Baer.pdf](http://www.advgrouptheory.com/GTArchivum/Baer/Baer.pdf)

So you won't take it amiss of me if I begin by describing my vision of a mathematician, so that I can tell you how I wished that I were. You will then comprehend the difference between Dream and Reality, and seeing the old teacher depart be willing to forgive much with an understanding smile.

It was ever my desire to realise in some form or other Dürer's engraving of "Saint Jerome in his study". He sits quietly reading in his study, faithfully protected from the world by a lion at his feet. Of faithful guardians I have had my own, mostly female and you will know of whom I speak. But whilst reading in my study I was rarely seated. Mostly, I laid more dreaming, either on a hill slope in summer or in bed at night. Sometimes the dreams yielded something, but sadly were unable to withstand the ruthless clarity that paper and typewriter demanded, and vanished again into the air from whence they came. However, one or other did become reality and the joy of achievement was enriched by the vain observation that as a result of my dreaming some mathematical text-books were improved. But those who carefully watched over me know full well that dreaming instead of reading was not my only dissimilarity with Saint Jerome. At times "Saint Jack of all Trades" would have been a more fitting title. I edited journals, even founded one; I wrote memoranda and organised conferences. The quite dreamer had become a busy administrator, the mortal sin of mathematicians, researchers and professors.

The excuses are many; one does it for one's students for whom one wished to provide the proper conditions for their work and development. All the more so, since one had formed them in one's own image (much to the eternal amusement of Mephistopheles). But only one excuse is really acceptable: what fate had placed in one's cradle was insufficient to do more. Thanks to the fate we could dream a plenty; often the sun shone for us, the blue sky arched the mountains and triggered a flash of inspiration that yielded new results. This zealous activity had its rewards, for it was precisely the Frankfurt decade that brought such an unexpected circle of scholars — scholars kind enough to have me believe that I could be of assistance to them and whom I may have influenced in some way or other; perhaps the pleasure of our work, perhaps my pedantry, my contempt for any importers feigning geniality, my utter disdain for those who consider mathematics merely as a business, a step on their way to power, an object for political and commercial gain.

My light-hearted dreamy portrayal of Mathematics fits naturally into my own perception of the subject: “Das Nachdenken über das Denken” — the process of Reflection on Thought. Certainly we do not reflect on all phases of thought. As an object worthy of our consideration it needs to be precisely formalised with a pointed sharpness that will enable our own thought processes comprehend it. For us there are no aesthetic judgements, no well-meaning transcriptions of a phenomenon that may only hinder our understanding of it solely by clarity of thought. For us the only antithesis of “true” is “false”. Yet we well know that the difference between good and bad mathematics lies somewhere in the aesthetic. On the opposite side lies the fear of everything, what Kant liked to describe as “Urteilkraft” — the power of judgement. Hence the anxiety linked to an appropriate not self-posed problem. It must have external approval. Hence the attention to famous problems. If Gauss (or any contemporary *ar-biter elegantiorum mathematicorum*) had posed the problem or could not solve it, then it must have been good. Thence the flight into the refuge of application, where the responsibility for our efforts is borne by another-one who applies our thinking. The resulting plunge into teleological vicious circles may not unduly bother such refugees who treat with disdain what they call Metaphysics. What is overlooked is that our judgement is taken away by demands that we simply produce applications for someone else to apply.

This transition from the aesthetic to the moral is serious. At least for us things are better. We can tackle a problem not because Paris says it is important, not because the master recommends it, but solely because it delights, fascinates and captivates us. But what are we to do with the fascinating problem when we have no idea where to begin?

The lucky ones will only engage problems they can handle. As Gantenbein put it “indeed the young researcher may have the better ideas, but the older one knows better what he can achieve”. I suppose it is no different in life generally, one may love and find his love is unrequited, another may find peace in his love.

If we reflect on thinking it may seem as if we reflected only on the thoughts of others. That may reek of application, but sometimes this is how it began. Early Greek pre-mathematics was mainly concerned with land measurement and interest-rate calculation. But even the general public would laugh heartily were I to proclaim that Geodesy was the “Real Geometry”.

The Greeks took the great and revolutionary step which removed the shackles of application from the godly image of Geometry. Even so, there come those who could not see beyond application; among them the all-conquering, world-plundering Romans. The Greeks presented the gift of Pure Geometry to the world.

The next giant step from abstract geometry to the discovery of other new possible geometries was only achieved in the last century — a discovery whose first faltering steps were hindered by the fearful bleatings of the Boeotians. But after Gauss, followed Riemann, followed Hilbert and came developments in several different and unexpected areas. Today we construct possible geometries, feasible number systems and new mathematics. Whether or not these discoveries find application is not our concern. The evidence of history proves that those least concerned with application deliver the applicable. Certainly it took many centuries before the abstract theory of conic sections, developed by the ancient Greeks, found application in astronomy. What I wish for those who absolutely desire to deliver something practical, I need not tell you. Nor do I need to point out to you that when we engage in Mathematics that “Nachdenken über das Denken” leads us not only to recognize, as Hilbert did, the ‘salto mortale’ risks we undertake but also that we are more essentially interested in thought-model construction and structural investigation.

Ever since the Holy Trinity of my mathematical generation, Dedekind, Hilbert and Emmy Noether, elevated this form of mathematics to the high point of our activity, ever since the steam-rolling Nicholas Bourbaki declared this mind-set to be the official dogma of Mathematics, one rarely hears a mathematical lecture, aimed at the general public, which does not reflect and celebrate the structural thinking. Again the weight of history comes in. Rather than plucking on a string with a differential equation the applied mathematician of tomorrow must either construct his own thought-models and structures or fetch them from the drawer of a thinker so that so-called realities can be understood and mastered.

This life in the clear air of pure thought, whose subject is again ideas, cultivates an unusual mathematical mentality. This mentality, not the substance of his daily reflections, poses its own peculiar problem for the mathematician.

For me *the* mathematician in Reinkultur has always been Hilbert. Perhaps not as he may actually have been in real life but as he came across in Hilbert folklore. Perhaps this Hilbert image represented the

dream-wish of an entire mathematical generation from privy councillors to failed geniuses. I found that the mathematicians mental attitude is most beautifully represented in the Andresen fairy tale — the king with no clothes. During a parade the king wore no clothes. However, anyone unable to see them was unfit for his office — so the story goes. Suddenly a child calls out “the king is naked!”.

Today we would speak of the child’s immaturity, that he had not yet outgrown puberty and of his lack of manners. Although the on-lookers may have conceded that perhaps he was right — but how tactless of him to challenge a taboo — especially those who wished to be granted higher positions and royal honours.

Now this child is the mathematician. He represents the mathematician’s challenge. All at once the dreamer and pure thinker becomes a being with a dangerous obligation. Socrates was also unable to see the king’s clothes and as a result drank from the hemlock cup.

For the time being we are better off. We are granted a fool’s pardon as long as we remain in the domain of pure thought and sign at most one unpopular appeal. It could be different for the next generation. This ruthless and unconditional mind-set which harbours harmless dreams of pure thought will become a self-evident mentality. This may have been the reason why Plato only permitted those with a knowledge of mathematics to enter his academy.

It brings about the mind-set and mentality that compels us to speak out fearlessly and tacklessly, as that child did, whenever we see that the king is naked.

It is easy to prescribe duties for others when one enjoys the good fortune of a fool’s pardon, when one, like Saint Jerome, can withdraw to his shell where a faithful lion stands guard and holds at bay the evil forces of an evil world. When one has reached the stage where further awards and honours are not required, it is easy to instruct others as to their obligations while he himself withdraws. But I have not imposed these duties. I believe that they are self-imposed and so one can hardly escape them.

Be that as it may, the old man goes. He leaves you behind with your dreams and problems in a world perhaps no more evil than any other. But we do have the ability to see evil more clearly, as mathematicians we see the king’s wickedness exposed. But be that also as it may, for me and my lions, the evening sunshine beckons where we can rest on the bench before the house, sitting and waiting

for you to come — bringing your dreams, your problems and above all news of your successes and telling of your life — a life we were part of and which now passes by faster and faster.

Zur freundlichen Erinnerung an die schönen und  
fruchtbaren dreiundzwanzig aktiven  
Frankfurter Semester.

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