On a problem of Sehgal on torsion units of integral group rings

G finite group

Zassenhaus Conjecture

Every torsion unit in $\mathbb{Z}G$ is conjugate in $\mathbb{Q}G$ to an element of $\pm G$

Proved if

* G nilpotent (Weiss)
* If G has a normal Sylow subgroup with abelian complement (Hertweck)
* If G is cyclic-by-abelian (Caicedo, Margolis, dR)

Problem Sehgal (N normal nilpotent subgroup of G)

u torsion unit of $\mathbb{Z}G$ that maps to one by the canonical $\mathbb{Z}G \longrightarrow \mathbb{Z}(G/N)$

Is u conj in $\mathbb{Q}G$ to an elemt. of G?

* True if $G = N$
* If N is a p-group or if u is a p-element. (Hertweck)

$a \in RG$, $a = \sum a_g g$ | $\varepsilon_g(u) = \sum_{x \in g^G} a_x$

Partial aug. of u at g

Marciniak-Ritter-Sehgal-Weiss

u is conj in $\mathbb{Q}G$ to an elemt of G

$\iff \varepsilon_g(u^x) \geq 0 \quad \forall x \in \mathbb{Z}$

HeLP

$$\mathbb{Z}G \longrightarrow M_n(\mathbb{Z}N)$$

$\mathbb{Z}N$ $\qquad n = [G:N]$

$$u \longrightarrow U$$

N nilpotent (Cliff-Weiss)
Matrix version of the ZC holds
for all n $\Leftrightarrow$ N has at most
on non-cyclic Sylow subg.

* Hertweck

$N_{p'} = A$ abelian, K cocyclic
subgroup of A

$$\sum_{x \in nK} |C_G(x)| \, \varepsilon_x(u) \geq 0$$